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A model of public and private partnership through concession contracts

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ABSTRACT

In this paper, we investigate the economics of concession under dynamic uncertainty using real option theory. We analyze the properties of concession as an instrument to privatize investment and management of public resources. In this context, we explore, in particular, three issues: (1) the conditions under which the contract is acceptable to both a public and a private party, (2) the conditions under which it is efficient, i.e. it is preferable to direct development and operation by the public sector, and (3) two different possible equilibrium solutions. Finally, we apply the theoretical results obtained to the case of a major public highway concessionaire in Italy.

Keywords: concession contract, real option, license, Autostrade per l'Italia, private and public partnership

JEL codes: D45; H44; H54.

CONTENTS

1 INTRODUCTION

In recent years Governments of developed and developing countries have followed a popular approach to cope with more stringent budget constraints and the constitutional unfitness of public administration to handle construction and management ventures. By attracting private firms and capital to manage businesses such as water, natural gas, telecommunications, electricity system and transport infrastructures, they have tried to achieve both higher economic efficiency and management accountability. In some cases, for instance in Argentina, Bolivia, Chile, New Zeland, UK, State owned companies have been privatized; in some other cases, such as in China, Colombia, India and Indonesia Governments have tried to construct new facilities under private initiatives without necessarily privatizing existing assets. Firms of regulated utilities in the US have sub-contracted many power stations. Different forms of private participation have been implemented, on the basis of various risk-sharing arrangements between investors, consumers and taxpayers (the "Government"). In some cases, private parties are allowed to operate a business but not to own it. In other cases, private parties also take on investment and financing risk as in the popular build-operate-transfer, BOT, model, through which a private company finances, constructs and operates a venture before turning it back to the Government.

Regardless of the differences in approach, a common concern surrounds most infrastructure privatizations. Many of the new private businesses, in fact, have monopoly power either because the market is a natural monopoly, and/or because Governments have given excusive rights to the private companies, thus instituting a form of legal monopoly. In this situation, to prevent private exploitation of monopoly power, it is often argued that private rights have to be limited so that the Government can take over the business or re-auction it to the private sector. This implies that the Government should have the responsibility to limit exploitation by private monopolists in the future as well.

In this paper, following Klein's (1998) definition, transfers of property rights, which Governments can limit in time and scope, are called concessions. These limits may be i) jointly fixed in advance by parties, ii) a function of the economic performance of the concession, iii) imposed by Government discretion, iv) or any mix of such rules. More broadly, Governments may wish to limit property rights so as to prevent appropriation of private monopoly power from holding up system development. For example, Governments may have rights to expropriate property owners.

The time limit itself is not the unique feature characterizing a concession contract. Licensing, for example, also contemplates a fixed term of validity. From a legal point of view, a concession differs from a licence because in the former case Government loses the possibility to use the public domain, while in the latter, Government simply allows a private concern to run a particular business, without losing any of its rights. Simple examples are the cases of the driving and haunting license on the one hand, and the concession to run a public beach, on the other hand. In spite of the straightforward legal distinction, in practice the terms “concession” and “licence” are used interchangeably. In the case of spectrum assignment, for instance, the public party definitely loses the possibility to use the portion of spectrum he assigns, hence one should refer to it as a concession, while it is general habit to refer to it as a license (see Klemperer (2002, 2002a), Van Damme (2002), Lee (2003), Maskin (2004), Scandizzo, Ventura (2006) and many others). Other forms of contracts between public and private entities, namely lease and management contracts, are closely related but differ from a concession in the rights of the operator and its remuneration. A lease gives a company the right to operate and maintain a public utility, but investment remains the responsibility of the public. Under a management contract the operator will collect the revenue only on behalf of the Government and will in turn be paid an agreed fee. Furthermore, according to Klein (1998) concessions differ fundamentally from contracts for equipment and civil works, because they establish a long lasting relationship and are more than a one-off purchase.

The remainder of the paper is organized as follows. Section 1 reviews the real option models applied to concession contracts. Section 2 put forward a baseline model to understand the relationships between public and private sector in a concession contract, Section 3 introduces the price variable in the model of Section 2, Section 4 applies the theoretical model to the case of the Italian toll motorways and, finally Section 5 summarizes the results and concludes.

2 REAL OPTIONS AND CONCESSION CONTRACTS

In practice, concessions are awarded through competitive auctions or, alternatively, by negotiations. Analysing costs and benefits of auctions *vs* beauty contests and, within auctions, costs and benefits of different formats[1] is far beyond our scope. We are interested instead in the study of economic relationships between public and private parties in a concession contract. More

1 In reality open auctions are less common for concessions that sealed bid. We refer the interested reader to Valletti (2001), McAfee R.P., McMillan J. (1987), Klemperer (2002a, 2002b).

specifically, we examine the problem of the properties of the economic properties of a concession contract under dynamic uncertainty. The lack of information about the future, in fact, creates a framework for negotiating the contract both as division of labor based on the different incentives of the two parties and as a risk sharing arrangement. Moreover, the irreversibility of infrastructure investment implies that the contract may be seen as a real option by both parties, so that the negotiation process can be interpreted as an exchange of contingent assets and liability over time.

The real options approach[2] allows us to look at the relationship between the Government and the private concessionary as a principal-agent contract reflecting both distributional and burden sharing considerations under the veil of uncertainty. With respect to the existing literature, this approach produces several new results. First, the concession institutes a bilateral bargaining game between the two parties, with an essential duopolistic structure. Second, the possibility of bargaining between the two parties is enhanced by uncertainty, since the additional dimension of risk sharing may be added to seek advantages for both parties. Third, both the cooperative and the Nash solution ensure that the contract is able to allocate resources efficiently, i.e as the Government would do if it implemented the project directly at private investment costs. Fourth, if the Government is able to act as a Stackelberg leader, however, the irreversible nature of the consequences of the investment, will complicate the consequences of uncertainty and will determine a loss of efficiency with respect to the alternative case of direct public development. Because of the fact that not only the states of nature, but also the performance of the agent are uncertain, in fact, under this class of solutions (the Stackelberg equilibria) the Government is induced to adjust twice for uncertainty, thus increasing the imputed cost with the square of the uncertainty/irreversibility factor. Fourth, as a consequence of this inefficiency, the Government should enter the contract only if its cost structure is sufficiently unfavourable as compared to private costs, i.e. if its investment costs are larger than private costs adjusted for the uncertainty on future cash flows. Finally, the introduction of a contract price does not change the cooperative and the Nash solution, but it causes further inefficiencies in the Stackelberg solution.

The attempt to model concession contracts through the real option framework is not new in the economic literature. Real options explicitly take into account aspects of some investments that are particularly suited to model concession contracts: the economic irreversibility of the investment (think of a railway, a water system, an oilfield, a toll road, etc.), uncertainty surrounding the expected returns and, of course, the expiration date of the economic enterprise.

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[2] For an introduction to real options theory we refer the interested reader to Micalizzi (1997), while for a complete, though dated, and more formal guide see Dixit and Pindyck (1994).

Some of the papers modelling the value of natural resources can be considered quite close to valuing the value of concession contracts, since the latter must necessarily be a function of the former. Tourinho (1979) first showed that resource reserves can be valued as options; in a very popular article, Brennan and Schwarts (1985) analyze interactions of operational options in a copper mine, but copper mines are exploited through concession contracts. More broadly, Schwartz and Trigeorgis (2001) present a number of applications of real option methodology to natural resources evaluation and Dias (2004) provides a concise review of real option models only for petroleum applications, considering that oilfield are usually explored and exploited through concession contracts.

Some papers modelling concession contracts aim to evaluate the value of the contract and/or the optimal timing to invest under different scenarios (Dias and Rocha (1999), Rocha, et al. (2001), Dias, Rocha and Teixeira (2004), D'Alpaos and Moretto (2004), D'Alpaos, Dosi and Moretto, (2006)), in which the value of the concession is nothing but the expected net present value of future payoffs, plus the option to delay the investment, should the concession design allow it.

Fewer contributions model the bargaining process between a public institution ("the Government") and a private party (the concessionaire) and the economic interactions leading to the agreement over a concession contract. Hung, Poudou and Thomas (2003) study the design of a concession contract under asymmetric information between the Government and several potential concessionaires, where production is in the depletion of a non–renewable resource. When the concession is awarded by auction, this mechanism leads to an efficient separation procedure among agents, so that the lowest cost agent is assigned the concession contract. This result can be viewed as an application of Laffont and Tirole (1987) analysis to exhaustible resources management problem when it is solved using incentive contracts. Once a concession is granted to a single concessionaire, in a bilateral monopoly, as compared to the outcome under symmetric information, Hung, Poudou and Thomas show that over- production is optimal at the end of the contract because the gains from exhaustibility dominate the rent lost under asymmetric information.

Still within the theme of optimal incentives, Meister (2005) argues that when arranging a concession bidding procedure and writing the contract, the public body has to consider investment incentives for the concessionaire. If the lifetime of the asset exceeds the contract term, the incumbent concessionaire faces a hold-up problem and underinvests. This phenomenon tends to be stronger in sectors where investment is very specific, long term oriented and hard to evaluate by a third party. Incumbent incentives may also vary when using different re-auction formats, tending to be the lowest in an English procedure and higher in a sealed bid (first or second price).

Dosi and Moretto (2007) consider the combinations between two different bidding rules and two different concession designs. The bidding rules consist alternatively in awarding the concession to the firm reporting the lowest price to consumers, or to the bidder offering the highest bid. While the first concession design imposes the obligation to immediately undertake the investment required to roll-out the service, the second gives the winning bidder the flexibility to optimally decide the investment time. The authors find that the two auction formats involve the same outcome in terms of price to consumers and concession fee when the contract rules out investment time flexibility. Moreover, this outcome is equal to the one that would emerge if the Government awarded a contract which did not impose the obligation to immediately operate the service by using the highest fee format. Consequently, Government choice reduces to the alternatives between imposing the obligation to invest immediately (in this case the bidding rule is irrelevant), or allowing the winning bidder to delay the investment, awarding the concession by using the lowest price format. This last alternative, also involves the highest expected welfare.

3 THE CONCESSION CONTRACT AND THE DEVELOPMENT VALUE

We model the case of a natural resource owned by (or under the regulatory power of) a public institution. The resource can be developed by the Government upon the commitment of investment costs concentrated in the first period (the "zero" period). Development yields a net cash flow formed by a systematic part, which is normalized to unity, and a stochastic part, denoted by y, observable in every period, even in the absence of development, evolving according to a stochastic process of the geometric, Brownian motion variety:

$$(1) \quad dy = \alpha y dt + \sigma y dz$$

where α and σ^2 are respectively the drift and variance parameters and dz is a normally distributed random variable such that $Edz=0$ and $Edz^2 = dt$.

We consider the class of valuations of the type:

$$(2) \qquad V(y) = \sup E_y \left[e^{-\rho\tau} \left(\int_{\tau+T}^{\infty} e^{-\rho(s-\tau)} y_s ds - C \right) \right]$$

where ρ is the discount rate of the decision maker and *T* is the time at which the stochastic benefit from development starts accruing to the decision maker. The basic case, examined in detail in the literature (see, for example, Dixit and Pyndick, 1994) is the valuation for $T = 0$. In this case the problem in (2) has the solution:

(3a) $$V(y) = y^* /(\rho - \alpha) - C \qquad y \geq y*$$

(3b) $$V(y) = (y^* /(\rho - \alpha) - C)(y / y^*)^{\beta_1} \qquad y < y*$$

where:

(4) $$y* = \frac{\beta_1}{\beta_1 - 1} C$$

is the optimal exercise boundary at which the investment opportunity should be exercised[3] and $(y / y^*)^{\beta_1} = Ee^{-\rho\tau}$ is the expected value of a discount factor that depends on the stochastic time of entry τ.

Because infrastructure projects typically depend on development of natural resources, such as real estate, landscapes, parkland, agricultural the undeveloped resource generates a flow of amenities that would disappear with development. This implies that the Government, in deciding on investment adoption, faces both direct and indirect (opportunity) costs. We can simply characterize the cost *C* for the Government with the reduction of a flow of amenities $\frac{x}{\rho}$ with social, but not necessarily private value (a positive externality of the resource that would be lost by developing it), and direct investment costs I_p. Equation (4) can thus be interpreted as stating that the threshold value of y where it pays to exercise the option to develop depends on indirect costs $-\frac{x}{\rho}$, direct investment costs I_p, and β, which is a function of the uncertainty of future income flows:

(5a) $$V_d(y) = \frac{y_d}{\delta} - \frac{x}{\rho} - I_p \quad \text{if} \quad y \geq y_d$$

(5b) $$V_d(y) = (\frac{y}{y_d})^{\beta_1} [\frac{y_d}{\delta} - \frac{x}{\rho} - I_p] \quad \text{if} \quad y < y_d$$

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[3] The value of the option to invest (see Dixit and Pindyck, 1994, p.140) is given by the expression $F(y) = A_1 y^{\beta 1} + A_2 y^{\beta 2}$ where A_1 and A_2 are constants determined by boundary conditions and β_1 and β_2 are, respectively, the positive and the negative root of the characteristic equation: $\rho - \beta(\rho - \delta) - \frac{\beta}{2}(\beta - 1)\sigma^2 = 0$

(5c) $$\frac{y_d}{\delta} = \frac{\beta_1}{\beta_1 - 1}\left(\frac{x}{\rho} + I_p\right)$$

If one assumes that dynamics of the risk contained in the cash flow, *dz*, can be spanned by existing assets, the option in the hands of the Government can be evaluated by applying contingent claim valuation and $\delta=\mu-\alpha>0$, μ is the total expected rate of return, as suggested by the Capital Asset Pricing Model (CAPM), ρ represents the risk-less rate of return.

In (5), the optimal exercise boundary depends on: (i) the total costs generated by investment, and (ii) the value of the uncertain yield at which the opportunity to develop the resource (the option value) equals the net benefit from development minus the value of the next best alternative. Equations (5a) - (5c) imply that the owner does not exercise the option until the returns exceed the sum of its investment costs and the opportunity costs by a factor $\beta/(\beta-1)$ that depends on the degree of uncertainty.

Consider now the possibility that the resource is developed through a fixed term contract (concession) with a private developer and assume first that the own development alternative is not feasible. The public owner in this case would face the alternative between keeping the resource undeveloped and developing it at the cost of foregoing its fruits for a concession period *T*, to be determined contractually. We assume first that there is no contractual payment; and that the developed resource returns to the Government after the concession period without any further obligation on the part of the concessionaire. From that point onward, the Government enjoys the full benefits of the developed resource. The concession thus acts as a substitute for the investment costs.

Under these conditions, we can determine the value of the contract for the principal (the Government) as the solution to the following problem:

(6) $$V(y) = \sup E_y\left[e^{-\rho\tau}\left(\int_{\tau+T}^{\infty} e^{-\rho(s-\tau)} y_s\, ds - \int_{\tau}^{\infty} x e^{-\rho(s-\tau)} ds\right)\right]$$

that is, using (3a) and (3b):

(6a) $$V(y) = \frac{y_p}{\delta} e^{-\delta T} - \frac{x}{\rho} \quad \text{if} \quad y \geq y_p$$

(6b) $$V(y) = \left(\frac{y}{y_p}\right)^{\beta_1}\left[\frac{y_p}{\delta} e^{-\delta T} - \frac{x}{\rho}\right] \quad \text{if} \quad y < y_p$$

(6c) $$\frac{y_p}{\delta} e^{-\delta T} = \frac{\beta_1}{\beta_1 - 1}\frac{x}{\rho}$$

Thus, for any given length T of the concession, there is a threshold value y_p at which the principal will decide to enter the contract rather than leave the resource undeveloped. Conversely, for each value of *y,* there is a maximum period of concession time T that keeps development convenient for the principal. Solving expressions (6c) for T, for an arbitrary value of *y,* yields:

(7) $$T \leq \frac{1}{\delta}\log\left(\frac{\frac{y}{\delta}}{\frac{\beta_1}{\beta_1-1}\frac{x}{\rho}}\right)$$

Proposition 1. *If own development is not feasible, the maximum incentive-compatible concession time that the principal (Government) may grant to the agent, for any given (expected) value of the stochastic variable, depends (negatively) on the volatility of returns and (positively) on the ratio between the expected present values of the cash flows under the development and the non-development alternative.*

Comment. In the literature on real options, expression (6) is interpreted as a condition that the expected returns have to satisfy in order to exercise the option to adopt the investment. Given sufficient time, since the forward looking expectation depends on the current value of returns, condition (6) determines what is the "right time" to exercise the option. A time dependent contract like the concession, however, gives the possibility of determining the condition for the exercise of the option for a wider range of expected returns, by appropriately setting the duration of the contract. In a sense, the contract thus depends on the concession time T as a price-like variable. A larger T implies a longer period during which the principal foregoes the revenues generated by the resource development. Such a period is longer the greater the forecast of such a loss, relative to current income, and the smaller is its volatility. In other words, the length of a concession granted by the Government is inversely proportional to the uncertainty of future returns. Higher uncertainty implies that the Government runs the risk of losing a significant amount of the upside of investment returns. Thus, higher uncertainty translates into a higher value of the option held by the Government and therefore into higher returns required by the Government to forego the option and into a lower period of concession time. Note that there is no feasible time for the concession unless $y/\delta \geq \frac{\beta_1}{\beta_1-1}x/\rho$, that is, unless at least a concession of zero duration would be acceptable to the Government. Even when this condition is satisfied, however, as Table 1 shows, it takes very high values of the PV ratio to build up any significant concession time. Note also that if the value of the externality tends to zero, the threshold values in (6) and (7) tend, respectively, to zero and to infinity.

Tab. 1 Limiting acceptable values of concession length for the principal under alternative combinations of volatility and PV ratios

under the hypothesis of zero trend ($\alpha = 0$)

Volatility	Ratio between development and non development PV						
σ^2	1,1	1,2	1,3	1,4	2,0	3,0	50
0,1	0,33	0,71	1,05	1,38	2,92	4,69	16,90
0,2	0,24	0,62	0,97	1,29	2,84	4,60	16,82
0,3	0,16	0,54	0,89	1,21	2,76	4,52	16,74
0,4	0,08	0,46	0,81	1,13	2,68	4,44	16,66
0,5	0,00	0,38	0,73	1,05	2,60	4,36	16,58
1,0	-0,37	0,00	0,35	0,67	2,22	3,98	16,20

Consider now the concession holder. For her, the value of the concession can be expressed as the solution of the following problem:

(8) $$V_\pi(y) = \sup E_y \left[e^{-\rho\tau} \left(\int_\tau^T e^{-\rho(s-\tau)} y_s ds - I_\pi \right) \right]$$

where I_π indicates the investment costs that the concessionaire would have to undertake to enter the contract. The solution to problem (8) is:

(9a) $$V_\pi(y) = \frac{y}{\delta}(1-e^{-\delta T}) - I_\pi \quad \text{if} \quad \frac{y}{\delta} \geq \frac{y_\pi}{\delta}$$

(9b) $$V_\pi(y) = (\frac{y}{y_\pi})^{\beta_1} [\frac{y}{\delta}(1-e^{-\delta T}) - I_\pi] \quad \text{if} \quad \frac{y}{\delta} < \frac{y_\pi}{\delta}$$

(9c) $$\frac{y_\pi}{\delta}(1-e^{-\delta T}) = \frac{\beta_1}{\beta_1 - 1} I_\pi$$

Condition (9a) corresponds to the state of the world where the concessionaire enters the contract, while condition (9b) indicates that a larger value of the cash flow is needed to convince the private firm to accept the concession. Expression (9c) indicates the threshold of cash flow expected present value at which the private party will accept the concession. For an arbitrary value of y, solving (9c) for *T*:

(10) $$T \geq \frac{1}{\delta} \log([1 - \frac{\beta_1}{\beta_1 - 1}(\frac{I_\pi}{y/\delta})]^{-1}) = \frac{1}{\delta} \log\left(\frac{\frac{y}{\delta}}{\frac{y}{\delta} - \frac{\beta_1 I_\pi}{(\beta_1 - 1)}} \right)$$

Proposition 2. *The minimum time of the concession (development plus operations) of a contract acceptable to the concessionaire is proportional to the logarithm of the ratio between the expected value of the cash flow and the net expected surplus from the minimum entry point for an infinitely long concession.*

Comment. Note that expression (10) is equivalent to the expression found for the principal in (7), with the difference that the lower acceptable bound for the concessionary depends on private investment costs I_π rather than on the externality x.

Recalling the condition for the principal, we can write, by combining (7) and (10):

(11) $$\frac{1}{\delta}\log\left(\frac{\frac{y}{\delta}}{\frac{y}{\delta}-\frac{\beta_1 I_\pi}{(\beta_1-1)}}\right)\le T\le\frac{1}{\delta}\log[(\frac{\beta_1-1}{\beta_1})\frac{(y/\delta)}{(x/\rho)}]$$

implying:

(12) $$\frac{y}{\delta}\ge\frac{\beta_1}{\beta_1-1}(\frac{x}{\rho}+I_\pi)$$

Note also that if the conditions for entry (6c) and (9c) for both parties hold simultaneously, equation (12) will hold as an equality. We thus have:

Proposition 3. *A concession contract will be feasible, if and only if the expected value of the stochastic cash flow is greater than or equal to the threshold value that satisfies the value-matching condition of the public sector for own development at private investment costs.*

Comment. Inquiring on the existence of a bargaining interval is equivalent to ask whether a solution exists for the problem of valuing the resource from the joint point of view of Government and concession holder, in the hypothesis that resource development is chosen. We can write this problem as:

(13) $$V(y)=\sup E_y\left[e^{-\rho\tau}\left(\int_{\tau+T}^{\infty}e^{-\rho(s-\tau)}y_s ds-I_\pi-\frac{x}{\rho}\right)\right]$$

and, if development is chosen, the solution is:

(14) $$V(y)=\frac{y}{\delta}-I_\pi-\frac{x}{\rho}\quad\text{if}\quad\frac{y}{\delta}\ge\frac{\beta_1}{\beta_1-1}(I_\pi+\frac{x}{\rho})$$

Corollary. *A concession contract will achieve an optimal cooperative solution if its duration is such that both parties want to enter it at the point where the expected present cash flow equals the sum of external and private costs.*

Substituting the threshold value in (14) into the expressions for the duration acceptable to each party in (11), we see that both parties agree with a concession of length $T = \frac{1}{\delta}\log\left(\frac{\frac{x}{\rho}+I_\pi}{\frac{x}{\rho}}\right)$, which corresponds to the simultaneous solution of (6c) and (9c).

Consider now the Nash solution, i.e. the concession length that maximizes the geometric average of the payoffs of the two agents:

$$(15)\quad T = \arg\max\left[\left(\frac{y}{\delta}e^{-\delta T} - \frac{\beta_1}{\beta_1 - 1}\frac{x}{\rho}\right)^{w}\left(\frac{y}{\delta}(1-e^{-\delta T}) - \frac{\beta_1}{\beta_1 - 1}I_\pi\right)^{1-w}\right]$$

where $0 \le w \le 1$ is a weight reflecting the bargaining power of the Government. Differentiating with respect to T, equating to zero and solving for T, we obtain:

$$(16)\qquad T = \frac{1}{\delta}\log\frac{y/\delta}{w[\frac{y}{\delta} - \frac{\beta_1}{\beta_1 - 1}(I_\pi + \frac{x}{\rho})] + \frac{\beta_1}{\beta_1 - 1}\frac{x}{\rho}}$$

which substituted into either (6c) or (9c), will yield condition (12) as an equality.

Proposition 4. *The conditions for a Nash and a cooperative equilibrium coincide.*

Comment. Even though the cooperative and the Nash solution are different in terms of concession time, they achieve the same result in terms of timing, in the sense that the option to implement them will be exercised in both cases at the same level of the expected cash flow. This means that in both cases the same efficiency condition will be met, namely the proceeds from the investment will be expected to cover both public and private costs, even though any deviation from this “value matching” condition will be distributed in a different way under the two hypotheses.

Given that condition (14) is satisfied, what are the characteristic of the best response[4] contract if the Government has the advantage of the first move and can credibly commit itself to one strategy? In other words, we assume that the conditions of a Stackelberg competition hold, i.e. the Government can act as a leader by committing itself under the ex ante knowledge that the follower (the private party) observes its action and has no means to commit herself to any

[4] The **best response** is defined as the strategy which produces the most favorable immediate outcome for the current player, taking other players' strategies as given (Fudemberg and Tirole, 1991, p. 29).

non Stackelberg follower strategy. Under these conditions, in order to explore the problem of the existence of a Subgame Perfect Nash Equilibrium (SPNE), we can proceed by backward induction and start with the problem of the follower:

(17) $$V_{\pi}(y) = \sup E_y \left[e^{-\rho\tau} \left(\int_{\tau}^{T} e^{-\rho(s-\tau)} y_s ds - I_{\pi} \right) \right]$$

According to (17), faced with the leader's proposal of a contract of length T, the follower will react by choosing an entry level for the expected cash flow that makes the expected value of her net earnings as large as possible. This implies that the threshold level for entry will be:

(17a) $$\frac{y_{\pi}}{\delta}(1 - e^{-\delta T}) = \frac{\beta_1}{\beta_1 - 1} I_{\pi}$$

Thus, the follower best response is to accept the contract for a value of the expected cash flow $y \geq y_{\pi}$ in (17a). Given the follower's best response, the leader solves the problem:

(18) $$V_p(y) = \max_T \left\{ \sup E_y \left[e^{-\rho\tau} \left(\int_{\tau}^{\infty} e^{-\rho(s-\tau)} y_s ds - \frac{x}{\rho} - \frac{y_{\pi}}{\delta}(1 - e^{-\delta T}) \right) \right] \right\}$$

For any given T this problem has the solution:

(18a) $$\frac{y_c}{\delta} = \frac{\beta_1}{\beta_1 - 1} \left(\frac{x}{\rho} + \frac{\beta_1}{\beta_1 - 1} I_{\pi} \right)$$

Proposition 5. *The threshold value at which the principal and the follower will both enter a concession contract whose length is determined as the SPNE of a Stackelberg game in which the Government is the leader, equals the value of external costs adjusted for uncertainty plus private investment costs adjusted twice for uncertainty.*

Imposing $y_c = y_{\pi}$ and substituting (18a) into (17a) we find the minimum concession length that the follower is willing to accept to enter the contract at the same value of expected cash flow at which it is optimal to implement the contract for the leader. Thus, this length of time is also the maximum duration for the concession to which the Government should be willing to commit on an ex ante basis:

(18b) $$T = \frac{1}{\delta}\log\left(\frac{\frac{x}{\rho} + \frac{\beta_1}{\beta_1 - 1} I_\pi}{\frac{x}{\rho} + \frac{I_\pi}{\beta_1 - 1}}\right)$$

Note that this value is lower than the value of the duration in the general Nash equilibrium case:

(18c) $$T = \frac{1}{\delta}\log\left(\frac{\frac{x}{\rho} + \frac{\beta_1}{\beta_1 - 1} I_\pi}{\frac{x}{\rho} + \frac{I_\pi}{\beta_1 - 1}}\right) < \frac{1}{\delta}\log\left(\frac{\frac{x}{\rho} + I_\pi}{\frac{x}{\rho}}\right)$$

where the RHS of the inequality indicates the duration of the contract in the Nash equilibrium case.

Corollary. *The NPSE duration of a concession contract in which the Government is the leader of a Stackelberg game, is always shorter than the duration of a correspondent contract where both parties interact under parity conditions.*

Comment. These results shows that if the Government behaves strategically, and has sufficient information on private behavior, a private-public partnerships may carry an intrinsic inefficiency in the form of a double adjustment for uncertainty. Private investment costs, in fact, are adjusted twice for uncertainty because the expected value of the concession to the concessionaire equals her investment costs multiplied by the uncertainty factor depending on the waiting option on the investment implied by the concession. In the Nash solution, the private party is compensated for uncertainty by the length of the contract, but in the Stackelberg case, the Government can obtain more by shortening the duration and ensuring that the private party is compensated for the risk undertaken by a sufficiently large value of the expected cash flow. Thus, the Government is able to choose a contract of a shorter duration and a longer waiting period, which is, on the whole more convenient from its point of view, but overall less efficient. Note that in this contract both conditions (11) and (12) are satisfied. Condition (11), on the one hand is satisfied because we have applied (15), i.e. we have assumed that the time of the contract is such as to convince the concessionaire to enter the contract at the expected PV decided by the principal. Condition (12), on the other hand, is satisfied because the minimum value that would convince the principal to enter the concession contract is larger than the minimum value that would be required in the case of own development at private development costs. In the concession, in fact, the principal is implicitly charged, not only straight investment costs, but also the costs corresponding to the uncertainty faced by the concessionaire. However, the increase in the expected cash flow obtained by waiting is shared by the two parties only up to the point were risks of the follower are covered, while any surplus beyond that point is appropriated by the leader. Because the principal has again to add a charge for uncertainty to her direct (the externality) and indirect (the concessionaire implicit charge) costs, in this case the concession is thus more efficient than own development only if the public sector is sufficiently inefficient, i.e. if public direct investment costs are sufficiently larger than private costs to counterbalance the additional charge for uncertainty.

If the Government has also the alternative to develop the resource directly, therefore, it should choose the concession only if the corresponding contract value (see expression (18)) is larger than the value of own development, i.e. if :

$$(19)\ \sup E_y\left[\left(\int_\tau^\infty e^{-\rho(s-\tau)} y_s ds - \frac{x}{\rho}\right) - \frac{\beta_1}{\beta_1 - 1} I_\pi\right] \geq \sup E_y\left[\left(\int_\tau^\infty e^{-\rho(s-\tau)} y_s ds - \frac{x}{\rho}\right) - I_p\right]$$

Because from the point the view of the Government, the Satckelberg solution under its leadership is the best of all possible outcomes, this implies that the concession is attractive only if $I_p > \frac{\beta_1}{\beta_1 - 1} I_\pi$.

Corollary. *The concession contract should be preferred to own development if and only if public investment costs would be larger than private investment costs adjusted for uncertainty.*

Comment. The concession contract may allow the Government to lower development costs by accessing a pool of more efficient resources through the private sector. On the other hand, because the contract requires decentralizing decision making, the concessionaire will find herself operating under dynamic uncertainty and will raise her costs correspondingly. Aside from any other form of transaction costs, therefore, dynamic uncertainty creates an additional contractual cost which will render the concession contract less attractive and potentially less remunerative, even if in principle private development costs are lower than public costs. Note that this additional cost would be associated and compounded with any subsequent contracting arrangement between the main concessionaire and possible sub-concessionaires. A sequence of subcontractors could very well dissipate any efficiency gain from privatizing resource development and use.

4 ON THE RELATIONSHIP BETWEEN TIME AND PRICE IN A CONCESSION CONTRACT

So far the concession contract has been assumed to be priceless. Introducing a price asked by the s

Government for the contract, eq. (6) becomes

(20) $$V(y) = \sup E_y \left[e^{-\rho\tau} \left(\int_{\tau+T}^{\infty} e^{-\rho(s-\tau)} y_s ds + P_M - \int_{\tau}^{\infty} x e^{-\rho(s-\tau)} ds \right) \right]$$

where P_M is, for each admissible value of the cash flow *y*, the minimum acceptable price to the Government and equivalent expressions to (6a)-(6c) and (7) are derived going thorough the same steps as before. We thus have:

(20a) $$V(y) = \frac{y_p}{\delta} e^{-\delta T} - \frac{x}{\rho} \quad \text{if} \quad y \geq y_p$$

(20b) $$V(y) = \left(\frac{y}{y_p}\right)^{\beta_1} \left[\frac{y_p}{\delta} e^{-\delta T} - \frac{x}{\rho} + P_M\right] \quad \text{if} \quad y < y_p$$

(20c) $$\frac{y_p}{\delta} e^{-\delta T} = \frac{\beta_1}{\beta_1 - 1} \left(\frac{x}{\rho} - P_M \right)$$

which in turn implies

(20d) $$P_M \geq \frac{x}{\rho} - \frac{y}{\delta} e^{-\delta T} \frac{\beta_1 - 1}{\beta_1}$$

The value of the contract to the Government is non decreasing in a positive price paid by the concessionaire, and correspondingly the value of the entry threshold in (6c) is decreasing with it.

By the same token, for the private party the maximizing problem in (8) becomes

(21) $$V_\pi(y) = \sup E_y \left[e^{-\rho\tau} \left(\int_{\tau}^{T} e^{-\rho(s-\tau)} y_s ds - I_\pi - P_m \right) \right]$$

where P_m is the maximum price the private party is willing to pay and equations equivalent to (8a)-(8c) are obtained simply by adding P_m to I_π.

(21c) $$\frac{y_\pi}{\delta}(1 - e^{-\delta T}) = \frac{\beta_1}{\beta_1 - 1}(I_\pi + P_m)$$

which, in turn implies

(21d) $$P_m \leq \frac{y_\pi}{\delta}(1 - e^{-\delta T}) \frac{\beta_1 - 1}{\beta_1} - I_\pi$$

Paying a price, the value of the contract to the concession holder decreases and, correspondingly, the entry threshold increases.

Going thorough the same steps for the feasibility condition in terms of time span, equation (11) now becomes

(22) $$\frac{1}{\delta}\log([1-\frac{\beta_1}{\beta_1-1}(\frac{I_\pi+P_m}{y/\delta})]^{-1}\le T\le\frac{1}{\delta}\log[\frac{\beta_1-1}{\beta_1}\frac{(y/\delta)}{(x/\rho+P_M)}]$$

implying

(23) $$\frac{y}{\delta}\ge\frac{\beta_1}{\beta_1-1}\left[\frac{x}{\rho}+I_\pi+(P_m-P_M)\right]$$

that is definitely not less than (12) since P_m is at least equal to P_M in the feasible region.

Proposition 6. *The bargaining space for the time span of the concession is non empty if the expected present value of the cash flow is sufficient to cover, adjusted for uncertainty, external and private investment costs plus any surplus paid by the concessionaire over and above the minimum price the Government is prepared to accept.*

Comment. The introduction of a price in the contract makes possible to use two instruments (duration and price) to cover the costs and account for uncertainty. Thus, any disagreement between the Government and the concessionaire on the duration of the contract may be resolved by the payment of an indemnity. Alternatively, the two parties may agree on the duration, but not on the starting date and again the price can be used to reconcile the differences. Note that if the two parties agree on the price to be paid ($P_M = P_m$), the threshold level and the optimal duration for the contract are again those of the cooperative solution.

As before, the Government, if well informed on the parameters of the game, may act as a Stackelberg leader and the concessionaire as a follower. In this case, by using the same technique used to obtain the result in (17), we obtain the SPNE solutions:

(24a) $$\frac{y_\pi}{\delta}(1-e^{-\delta T})=\frac{\beta_1}{\beta_1-1}(I_\pi+P)$$

where

(24b)

$$T=\arg\max V_p(y)=\arg\max\left\{\sup E_y\left[e^{-\rho\tau}\left(\int_\tau^\infty e^{-\rho(s-\tau)}y_s ds-(\frac{x}{\rho}-P)-\frac{\beta_1}{\beta_1-1}(I_\pi+P)\right)\right]\right\}$$

(24c) $$\frac{y_{cc}}{\delta}=\frac{\beta_1}{\beta_1-1}[(\frac{x}{\rho}-P)+\frac{\beta_1}{\beta_1-1}(I_\pi+P)]$$

Proposition 7. *The threshold value at which the principal and the follower will both enter a concession contract whose length is determined as the SPNE of a Stackelberg game in which the Government is the leader equals the value of external costs minus the contract price adjusted for uncertainty plus the sum of private investment costs and the price adjusted twice for uncertainty.*

Comment. The introduction of a price increases further contract costs, since the value function of the principal, as shown in (24b), directly increases for the price paid, but also indirectly decreases since the same price is a cost that the concessionaire will have to recover together with the increased uncertainty that it will cause to her expected net gains. This increase in transaction costs can be measured by comparing the threshold cash flow y_c in (18a) with y_{cc} in (24c). By subtracting the former from the latter we obtain:

(25) $$y_{cc}-y_c=\frac{\beta_1}{(\beta_1-1)^2}P$$

Thus, the threshold will increase of a fraction (since $\beta_1>1$) of the price agreed and the increase will be larger the larger uncertainty[5].

Consider now the Nash solution. The Nash price can be determined maximizing the product of the player's net payoffs. Deriving the net payoffs of the Government, Π_s, and the private party, Π_p, respectively from (20c), and (21c), i.e.

$$\Pi_s(P)=\frac{y}{\delta}e^{-\delta T}\frac{\beta_1-1}{\beta_1}+P-\frac{x}{\rho}$$

for any possible value of P, and

$$\Pi_p(P)=\frac{y}{\delta}(1-e^{-\delta T})\frac{\beta_1-1}{\beta_1}-I_\pi-P$$

the Nash equilibrium price is

5 Taking the derivative of the RHS of (25) with respect to β_1 yields: $-\frac{\beta_1^2-1}{(\beta_1-1)^4}P$. This derivative is less than zero. Thus, since $\frac{\partial\beta_1}{\partial\sigma}<0$, it follows that the difference between the threshold with and without the price increases with uncertainty if the price is positive.

(26) $$P_R = \arg\max_p \left\{ \left[P + \frac{y}{\delta} e^{-\delta T} \frac{\beta_1 - 1}{\beta_1} - \frac{x}{\rho} \right]^w \left[\frac{y}{\delta}(1 - e^{-\delta T}) \frac{\beta_1 - 1}{\beta_1} - I_\pi - P \right]^{1-w} \right\}$$

(27) $$P_R = w\left[\frac{y}{\delta} \frac{\beta_1 - 1}{\beta_1}(1 - e^{-\delta T}) - I_\pi \right] + (1-w)\left[\frac{x}{\rho} - \frac{y}{\delta} e^{-\delta T} \frac{\beta_1 - 1}{\beta_1} \right]$$

where w is the Government bargaining power. Substituting the equilibrium price in (27) in either equations (20d) or (21d), we obtain again condition (12).

Proposition 8. *The threshold below which the bargaining interval (both in terms of time and price) becomes empty coincides with the entry threshold generated by a Nash equilibrium price.*

Comment. The feasible region in Proposition 3 and 8 has been derived by combining a twofold maximizing process carried out by the public and the private party, from the time and the price perspective. The maximizing process in (26) can be regarded as a joint maximizing process that, conditioning on the parties' net payoffs, as derived just from (20) and (21) finds the equilibrium price. Therefore, constraining the price to be such that the bargaining interval is not empty, reflects the feasibility condition expressed in (12).

5 AN EMPIRICAL APPLICATION OF THE MODEL

Autostrade per l'Italia (AI) is a company held by Atlantia, formerly Autostrade S.p.a., that owns 100% of its common stock. It is Italy's largest toll motorway builder and operator, with more than four millions daily customers, operating under a concessionary regime expiring on 31 December 2038 as established in the agreement drawn up with the assignor ANAS in 1997. AI is the largest private investor in infrastructure in Italy with over €18 billions of projects major work.

Recently, the company has been involved in a dispute with the Italian Government for not having completed the investment plan agreed under the ongoing concession contract. This agreement, drawn up in 1997, provided for an investment plan to be undertaken by AI for a total of €4.4 billions. A subsequent agreement, drawn up in 2002, provided for €5.9 billions of additional investment. In 2007 the commitments had been fulfilled only for 31% and 9%, respectively (Atlantia, 2007). The company, however, disclaimed any responsibility and attributed the delay to bureaucratic authorization not received, or not received by a reasonable date, from the Government, which in turn attributed the delay to conscious efforts by AI to defer the financial commitments. The dispute was eventually settled by the EU Commission and in October 2007 the parties came to a new agreement, whereby AI committed itself for a total of additional investment of €7.7 billions to carry out over the period 2008-2020.

While the legal details of the dispute are beyond the scope of this paper, our analysis aims to provide a test of whether the figures the parties agreed on were economically sustainable both for the company and the Government. More specifically, we can test whether at the time of the agreements the real options corresponding to the investment commitments were or not in the money, and under the hypothesis of a Stackelberg equilibrium, we can infer the increase in threshold value necessary to induce the concessionaire to enter the contract paying the price corresponding to the investment level provided by the agreement.

5.1 The investment options

A first experiment we can perform applying the model is to test whether eq. (12) is fulfilled at the different points in time the covenants were signed, in the sense that the expected cash flow from the concession was in the feasible region, as defined in Proposition 8. Because estimates of α and σ from historical data are not possible given the changes in accounting criteria over time, we chose to perform Montecarlo (MC) simulations based on Brownian motion parameters determined from reasonable assumptions and intervals derived from real options theory. In particular, the drift parameter α and $\delta=\mu-\alpha$ both belong to the narrow interval (0; μ), where μ is the total expected rate of return, as suggested by the Capital Asset pricing Model[6] (CAPM). We can then approximate μ using current estimates of CAPM between the returns on equity of AI and the returns of S&P 500 (Yahoo Finance provides an estimate of *"b"* at 0.688 http://it.finance.yahoo.com/q/tt?s=ATL.MI). Combining this information with data reported in the appendix, δ is numerically defined in the interval $\delta\in(0; 4.14\%)$.

Consistently with the estimate of μ, and therefore of δ, it is also possible to work out an approximate upper bound for the variance of the process. We know

6 We recall that $\mu = \rho + b(r_m - \rho)$ where r_m is the expected rate of return of the market and b is given by the covariance between market returns and the cash flow of the project divided by the variance of market returns. Differently from usual notation we use the symbol "b", instead of β for the covariance to avoid confusion with β, the roots of the characteristic equation in footnote 3.

that $b = \frac{\sigma_{my}}{\sigma_m^2}$ where *m* and *y* respectively represent the market and the project returns, therefore $\sigma_m b = \frac{\sigma_{my}}{\sigma_m}$ and by the Schwarz inequality $\sigma_m b = \frac{\sigma_{my}}{\sigma_m} \leq \sigma_y$, where σ_y represents the volatility of the returns to the project, the approximation of σ[7]. As the volatility of returns on equity prices is greater than the one on cash flow, because the former is largely affected by speculative trading, specially for a company listed, we can consider the estimate of the volatility of the return on equity prices as an upper bound of the volatility of the cash flow: $\sigma \in [b\sigma_m;\sigma_e)$ where σ_e stands for the volatility of the equity, numerically σ∈[10.23%; 17.06%).

Once defined the intervals for δ and σ, the MC simulations have been performed by attaching to each interval a probability density function (pdf) with the probability weights concentrated on three points of the domain: the lower bound, the mid point and the upper bound. For each experiment 10,000 random values have been generated for the values of the entry thresholds.

The intervals for y_e reported in column 4, indicate the likely interval consistent with the values that α and σ take on in column 2. Therefore, if the true value of y falls within one of the intervals, one can claim that the investment option was in the money for those given values of α and σ. A true value of y_e lower than the minimum lower bound (182.04 and 241.26 row 2 column 4 respectively for 1998 and 2005) points out to the fact that the option was not exercised at the money for any plausible value of the process parameters. Conversely, should the operating cash flow take on a true value greater than the highest possible upper bound (313.20 and 415.07 row 3 column 4 respectively for 1998 and 2005), we may conclude that the investment options were in the money for any admissible value of α and σ.

The most prominent commonality shown in Table 2 is that both investment options, in 1998 and 2005, the first year the covenants were in force[8], were deeply in the money. From AI financial statements, in fact, operating cash flow turns out to be respectively 585.97 and 889.668 millions of euro in 1998 and

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7 The approximation consists in the fact that "b" provided by Yahoo Finance is estimated using the returns on S&P 500 and the returns on equity, while a correct practice should have correlated the former to an appropriate measure of the projects' return, such as the operating cash flow. However, this approximation does not alter the results in any way and it is in line with the empirical literature, a similar strategy has been adopted by Schwartz (2004).

8 As a starting point for the investment plan we refer to the first year the contract is in force. For the 2002 agreement it took three years in order to get all the necessary bureaucratic authorization from the Government. Another possibility is to refer to the same year the agreement is signed. In this case, the conclusion about 2002 agreement does not change at all, being the operating cash flow in that year 698,221 millions, but nothing can be said for 1997 agreement since there are no available information.

2005. For any combination of the parameters, the upper bound of the simulated intervals is always less than the true value of the operating cash flow earned by the firm. In terms of Proposition 8 this result shows that when the covenants were signed the value of the cash flow was in the feasible region, consistently with a Nash equilibrium price.

Tab. 2 Montecarlo simulations for y_e at t=1998 and t=2005 (millions of Euro)

Distribution of α and σ	Mean of the distrib of δ and σ	Point estimate y_e 1998; y_e 2005	Range of simulated y_e. 1998 2005	Range width w.r.t. point estim.	Mean 1998 2005	Var 1998 2005	Skewness 1998 2005	Kurtosis 1998 2005
Normal	Both at lower bound	182.047 241.261	[182.04: 191.21] [241.26: 253.41]	5.03% 5.03%	184 244	2 4	1.0 1.0	3.86 3.86
Normal	Both at mid point	231.247 306.464	[204.25; 261.63] [270.69; 348.06]	22.05% 22.05%	231 306	51 90	0.07 0.07	303 3.03
Normal	Both at upper bound	312.379 413.987	[248.79; 313.20] [329.71; 415.07]	20.62% 20.62%	296 392	86 151	-0.66 -0.66	3.25 3.25
Normal	δ lower bound, σ mid point	202.709 268.644	[186.35; 219.01] [246.97; 290.25]	16.11% 16.11%	203 269	20 35	0.01 0.01	3.01 3.01
Normal	δ lower bound σ upper bound	223.364 296.017	[197.23; 223.38] [261.38; 296.04]	11.71% 11.71%	218 289	15 26	-1.02 -1.02	3.91 3.91
Normal	δ mid point, σ lower bound	201.263 266.728	[192.39; 220.77] [254.97; 292.58]	14.10% 14.10%	204 271	14 25	0.40 0.40	3.23 3.23
Normal	α mid point, σ upper bound	231.247 306.464	[224.22; 271.60] [297.15; 359.40]	20.32% 20.32%	252 334	45 79	-0.37 -0.37	3.17 3.17
Normal	α upper bound, σ lower bound	246.465 326.633	[209; 261] [277; 346]	21.10% 21.10%	242 320	51 90	-0.62 -0.62	3.19 3.19
Normal	α upper, σ mid point	281.717 373.351	[232.052; 304.72] [307.53; 403.84]	25.80% 25.80%	273 361	98 173	-0.3 -0.3	3.08 3.08

The variance of the pdfs of α and σ has been set equal to $(x/10)^2$, where x represents the mode of the parameter distribution. In each cell the 2005 value is reported in the lower row.

The new investment plan signed in 2007 deserves somewhat more attention. By this covenant the company takes on the commitment to complete the old investment, for a total of 10,286 millions of euro plus a new one for about other 7,700 millions of euro, thus reaching an amount for the new investment plan of a total of 17,986 millions of euro.

As the 2008 cash flow is not available yet, one must think in terms of possible scenarios. In particular, it is useful to define two opposite working hypotheses: a "best" and a "worst" scenario. These are, respectively, the lowest and the highest interval for the operating cash flow required to exercise at the money the investment option. Since $\frac{dy_e}{d\delta} > 0$ and $\frac{dy_e}{d\sigma} > 0$ the best possible situation is met when the two parameters are at the lower bounds, conversely for the worst possible scenario.

The table below reports the simulation of the two extreme situations consistently with option theory and a total investment cost of 17,986 millions of euro

Tab. 3 Montecarlo simulation of extreme intervals for y_e in 2008

Distribution of α and σ	Mean of the distrib of δ and σ	Point estimate y_e 2008	Range of simulated y_e	Range width w.r.t. point estim.	Mean	Var	Skewness	Kurtosis
Normal	Both at lower bound	740.120	[740.10; 777.38]	5.03%	748	32	1.0	3.86
Normal	Both at upper bound	1269.990	[1011.45; 1273.33]	20.62%	1203	1421	-0.66	3.25

The variance of the pdfs of α and σ has been set equal to $(x/10)^2$, where x represents the mode of the parameter distribution.

A comparison between the 2006 value and the outcome of the MC simulations shows that the simulations work out quite reasonable figures. The 2006 operating cash flow, €1,077 millions, is not significantly different from the upper bound of the worst scenario, 1,273.33, third row, fourth column, namely it is quite close to the highest level of the cash flow required to exercise the option at the money. This finding supports the choice made by the company and the Government to agree upon such a high level of investment, it is consistent with the economic rational behind the model, and can be read in the sense that the parties have made a cautious decision in setting the investment cost at about 18 billions, because compatible with the worst scenario.

Given that the cash flow evolves according to a stochastic process, one might wonder whether there still is a positive probability that a downturn occurs by 2008 and the option is not exercised at the money by that time. Of course one cannot rule out this possibility, but since y follows a Markov process, the forecast of its relative increment is αdt, and in the scenario under scrutiny α is set at zero, therefore $E(y_{08})=y_{06}$ and the option is rationally expected to be exercised at the money even in the worst possible scenario drawn by the model.

Two points are noteworthy: first, as the investment figures are bargained over by the Government and the private party, it is rational to assume that the agreement is reached taking also into account the externality. We can thus interpret the figures as the sum of the two components (x/ρ+I). Second, other pdfs have been used in simulating the possible scenarios, such as the triangular and the uniform, but all the conclusions remain unchanged.

5.2 Comparison between the Nash and the Stackelberg solution

Which of the two equilibria presented in the model better describe the actual outcome of the covenant signed in 2007? Did the Government act as a leader, or rather the outcome was the result of a Nash equilibrium?

To answer these questions we adopt the following step-by-step strategy:

- calculate P;
- work out the values of ycc-yc as in (25) for different plausible values of α and σ; select the values obtained in point 2 consistent with the actual outcome;
- compare them to the results of the Nash equilibrium.

1. The 2007 agreement establishes that the concession fee is set at 2.4% of annual net toll revenues. Substituting the model lump-sum price with the expected discounted value of all the future annuities, and defining as R_t the net toll revenue that accrues to the firm at time t, we can rewrite the concession price P_R as $P_R = E_P \int_0^T P_s e^{-\rho s} ds = E_R \int_0^T \lambda R_s e^{-\rho s} ds$ where λ is the constant fraction of net toll revenue due to the Government. To keep things as simple as possible, we assume R_t to grow at a constant rate over time and we estimate it from the past values of R_t as the average growth rate. Under this assumption the concession price can be rewritten as $P_R = \lambda \int_0^T R_0 e^{-(\rho-\gamma)s} ds$ whose solution is $P_R = \lambda \frac{R_0}{\rho-\gamma}(1-e^{-(\rho-\gamma)T})$ where γ represents the constant growth rate of R_t, numerically γ=3.98% and P_R=1,213.55 millions of euro.

2. The values of y_{cc}-y_c as in (25) are reported in Table 4, according to different values of α and σ

Tab. 4 **Values of y_{cc}-y_c under different scenarios (millions of euro)**

	δ lower	δ mid point	δ upper
σ lower	204,991,085	3,205	592
σ mid point	254,227,341	4,688	1,043
σ upper	308,737,664	6,262	1,509

The results consistent with the outcome are reported in bolt.

3. Only two of the tabulated figures in Table 4 are consistent with the actual value of the cash flow in 2006, 1,077 millions of euro, and they are reported in bolt, 592 and 1,046 millions of euro, under the most favourable scenarios for the two parties characterized by a high drift rate coupled with low or medium volatility. Furthermore, from Table 4 it is possible to extract the additional information that introducing a price in a concession contract when the Government acts as a leader, makes the threshold value y_c increase by an amount ranging from 48.7% to 85.9% of the price, respectively for the high α and low σ and for the high α and mid σ, worked out as the ratio (y_{cc}-y_c)/P.

4. The Stackelberg solution is consistent only with two possible scenarios which are very favourable to the parties, while the Nash (or cooperative) equilibrium is consistent with any possible value of the parameters. On the one hand, we can infer that if the Government acted as a leader, the parties were relying on two extremely favourable states of the world. On the other hand, it is more likely that the covenant reached was the result of a Nash (or cooperative) equilibrium since the outcome is consistent with any possible value of the parameters, as shown in Table 3. Put another way, there are reasons to consider the Nash equilibrium as more likely, or better able to capture the real outcome.

It is straightforward that the exercise can be repeated for different levels of δ and σ, even with a Montecarlo simulation for y_{cc}-y_c, however, the purpose of this empirical application is to show that the theoretical model can have clear-cut applications and that its explanatory power is strong in reproducing actual results. The model can be a good tool both in the ex-ante stage, when parties are supposed to evaluate the expediency in making the deal, as a device to provide sensible figures to bargain on, and as a means for ex-post evaluation and interpretation of the economic events occurred in a concession contract.

6 CONCLUDING REMARKS

This paper has investigated a topic of increasing interest: privatizing the development and management of public resources through the institute of the concession. Casting this problem in the realistic framework of dynamic uncertainty and real option methodology we show that this type of contract will be efficient if private investment costs are lower than public costs and either a cooperative or a Nash equilibrium is reached between the two parties. However, if the power of the Government is such that it has the right of the first move, and it may act as a Stackelberg leader by credibly committing itself to a given contract proposal, the solution will not be efficient. In this case, in order for this form of privatization to be socially desirable, not only private costs are to be lower than public ones, but also they must be sufficiently lower to overcome a double adjustment for uncertainty. This result follows from the fact that the principal agent relation of the concession contract, if carried out under dynamic uncertainty, involves a special type of transaction cost due to the fact that both the concession holder and the concessionaire will carry the burden of the possible unfavourable outcomes. We also show that these results continue to hold if a concession price is paid by the concessionaire to the Government. This possibility does not change the result on the efficiency of the contract under either the cooperative or the Nash solution, but further increases the inefficiencies under the Stackelberg equilibrium.

Finally, an empirical application to the case of Autostrade Italiane (AI) suggests that the concession contract awarded by the Italian Government, meets all the limiting conditions established in the theoretical part of the paper and can be characterized as a near optimal bargain under the worst possible scenario.

Anecdotal evidence and interviews to AI managers point out that the Italian Government was not the first mover in the 2007 covenant, so that we can reasonably consider the covenant as a Nash equilibrium. Further, under the Italian legislation on public works, AI and the Government did not hold any special right for a “first move”, and bargaining was in fact encouraged by the institutional and political framework. The results obtained are thus not inconsistent with our theoretical findings.

APPENDIX

DATA AND ESTIMATES OF MODEL PARAMETERS

Parameter	Value	Source
μ total expected rate of return, or expected rate of return from owning the completed project	4.14%	Our estimates on Datastream, S&P 500 and Yahoo Finance data
r_m market rate of return	4.51%	Datastream. average return of S&P 500 composite index over the period 1998-2006
ρ risk-free interest rate	3.30%	Datastream average Italian zero coupon bond over the period 1998-06
β from the CAPM provided by yahoo finance	0.688	Yahoo Finance http://it.finance.yahoo.com/q/tt?s=ATL.MI
σ_m standard deviation of market rate of return	14.86%	Datastream. SD of S&P 500 composite index returns over the period 1998-2006
Y_{98} operating CF in 1998	€ 585.970 Millions	Balance sheet data available at http://www.atlantia.it/en/bilanci/
Y_{02} operating CF in 2002	€ 698.221 Millions	Balance sheet data available at http://www.atlantia.it/en/bilanci/
Y_{05} operating CF in 2003	€ 889.668 Millions	Balance sheet data available at http://www.atlantia.it/en/bilanci/
Y_{06} operating CF in 2006	€ 1,077.129 Millions	Balance sheet data available at http://www.atlantia.it/en/bilanci/
γ net toll revenue growth rate	3.98%	Average growth rate in toll revenues $\gamma=T^{-1}[\log(R_{06})-\log(R_{98})]$. Balance sheet data available at http://www.atlantia.it/en/bilanci/
R_{06}	€ 2,309.8 Millions	Net toll revenues in 2006. Balance sheet data available at http://www.atlantia.it/en/bilanci/
P_R lump sum concession fee	€ 1,213.55 Millions	Sum of discounted future concession fees based on the projection of current net toll revenues

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