



ISTITUTO DI STUDI E ANALISI ECONOMICA

## **THE DARK, AND INDEPENDENT, SIDE OF ITALY**

by

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## **ABSTRACT**

The aim of this paper is twofold. First, new annual data on Italian irregular sector for the period 1980-1991 are reconstructed. These data are compatible with the available 1992-2001 official data. Second, based on this self-consistent “long” sample a time series analysis of the two sides – the dark and the regular - of the Italian GDP is performed. Results from univariate and VAR models seem to suggest that there are no connections (causal relationships, feedbacks, contemporaneous cyclical movements, common stochastic trends) between these two time series. In this sense, we could correctly refer to the Italian black sector as an “independent economy”.

JEL Classification: C53, H26.

Keywords: Underground Economy; Unit Root, VAR models.

## NON TECHNICAL SUMMARY

The linkages between the recorded and the hidden economy are still largely unknown. Despite the obvious difficulty in gathering information about a “shadow”, there are several important reasons to analyse the potential links between the regular and the irregular side of the economy. In a highly indebted system, like Italy, may be useful to ask oneself if fiscal policy can go on with a long sequence of surpluses, hoping that the regular sector does not sensitively react. A “mass escape” from the regular sector would dramatically reduce government revenues worsening the public budget situation. Economic literature suggests that the tax rates are negatively associated both with the labour supply and/or, as recently pointed out, with the tax evasion. The linkages can derive from labour market policies as well. In a paper by Boeri and Garibaldi it is argued that any unemployment reducing policy will endogenously reduce shadow employment, while it is very difficult to reduce shadow employment without increasing unemployment. On the positive side, in a climate of economic stagnation and decline the underground economy may serve a useful economic and social function providing jobs to many of willing workers. In addition, from firms’ point of view, the black workers pool allow increasing the degree of flexibility, from the public finance point of view, to the extent policymakers can convert irregular incomes into regular ones, the underground economy could be seen as a resource rather than a constraint. The tax amnesties implemented in Italy during the last decades are suggestive episodes as regard to this possibility. Elaborating Istat data, I reconstruct a 1980-2001 annual time series for the non-observed GDP. This has been possible because despite the new system of accounts (ESA95), former (ESA79) estimates of the non observed sector are validly exploitable for a backwarding reconstruction of the new releases. As regard to the shadow figures, all that the new ESA95 implies seem a zero-sum re-allocation game. This “long-term” information allows a time series analysis of the relationships potentially linking the dark and the clear side of the Italian economic system. Robust univariate and bivariate statistical devices support the findings that the two sides seem to have no connections – no causal relationships, no feedbacks, no contemporaneous cyclical movements, no common stochastic trends. Thus, one stylised fact featuring the hidden economy is its independence from the official one.

# **LA FACCIA NASCOSTA – E INDIPENDENTE – DELL’ECONOMIA ITALIANA**

## **SINTESI**

L’argomento di questo articolo è l’analisi delle relazioni tra l’economia sommersa e quella ufficiale nell’Italia del ventennio 1980-2001. Anzitutto, sulla base di dati Istat, si ricostruisce per il periodo 1980-1991 una serie storica del PIL irregolare da collegare alla corrispondente stima dell’Istat disponibile per gli anni 1992-2001. Poi si segue un approccio puramente statistico (sia univariato che bivariato) volto a definire alcuni fatti stilizzati che legano i due lati, quello oscuro e quello regolare, del PIL. I risultati, resi robusti da numerosi e complementari esperimenti, sono sorprendenti poiché sembrano indicare l’assenza di legami tra i due settori. Di qui il senso dell’aggettivo “indipendente” proposto nel titolo.

Classificazione JEL: C53, H26.

Parole chiave: Economia sommersa; analisi univariata, modelli VAR.

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## 1. INTRODUCTION

The non-observed sector of the economy has neither a commonly accepted definition, nor a commonly used name. A plethora of terms (underground, subterranean, moonlight, hidden, irregular, shadow, black, etc.) have been used to call it. All of them are suggestive of a particular aspect of the phenomenon, which is manifold. I will indifferently use here some of these adjectives but, in the Italian case, the most appropriate one turns out to be “independent”. Since I use data drawn from the Italian national institute of statistics (Istat), the definition of the black economy is the “official” one. Thus, the hidden production here studied represents (SNA, 1993) the area of (legal) production activities that are not directly observed due to reasons of economic nature (deliberate desire to avoid taxes and/or to avoid observing the law provisions concerning the labour market) and/or statistical nature (*e.g.* due to the failure to fill out the administrative forms or statistics questionnaires). Italian GDP contains both the “economic underground”, which is its irregular part, and the “statistical underground”, which is allocated to the regular (directly observed) GDP.

There are several important reasons to analyse the potential links between the regular and the irregular side of the economy. In a highly indebted system, like Italy, may be useful to ask oneself if fiscal policy can go on with a long sequence of surpluses, hoping that the regular sector does not sensitively react. A “mass escape” from the regular sector would dramatically reduce government revenues worsening the public budget situation. Economic literature suggests that the tax rates are negatively associated both with the labour supply and/or, as recently pointed out (Goolsbee, 1999), with the tax evasion. The linkages can derive from labour market policies as well. In a paper by Boeri and Garibaldi (2002) it is argued that any unemployment reducing policy will endogenously reduce shadow employment, while it is very difficult to reduce shadow employment without increasing unemployment. On the positive side, in a climate of economic stagnation and decline the underground economy may serve a useful economic and social function providing jobs to many of willing workers. In addition, from firms’ point of view, the black workers pool allow increasing the degree of flexibility (Signorelli, 1997; Bovi and Castellucci, 1999), from the public finance point of view, to the extent policymakers can convert irregular incomes into regular ones, the underground economy could be seen as a resource rather than a constraint. The tax amnesties implemented in Italy during the last decades are suggestive episodes as regard to this possibility.

To the best of my knowledge, very few works focusing on this topic with a medium-term perspective are available because of the shortage of reliable time-

series data (for an exemption see Bhattacharyya, 2004). The first aim of this paper is to compile a relatively long time series for the Italian hidden economy. The attempt is based on two annual data sets released by the Istat, one computed for the period 1992-2001 according to the new system of national account (Eurostat, 1995; henceforth ESA95), the other computed for the period 1980-1997 according to the previous system of national account (Eurostat, 1979; henceforth ESA79). Since the new definitions of the non-observed economy do not affect either the total GDP or the total full time equivalent (FTE) labour input<sup>1</sup> (Calzaroni, 2000), it turns out to be possible to generate a self-consistent decomposition of the regular and the irregular component of the Italian real GDP for the period 1980-2001.

Using these new data, I examine the relationship between unreported and regular GDP, to point out some stylized facts via a time series analysis. Missing a consolidated economic theory and to limit the curse of dimensionality, I chose to be as agnostic as possible. In other words, with a proper allowance for the stochastic properties of the data, several bivariate VARs are estimated. Then, impulse response functions with Monte Carlo based bands are computed in order to see if and how the two sides of the market interact. Somewhat puzzling, results show that the regular sector seems to be rather orthogonal to the dark side of the Italian economic system and, less univocally, *vice versa*. No Granger causality, no common stochastic trend, no contemporaneous movements, no shocks transfer from one market to another emerge from the data.

The paper is organised as follow. The next section deals with the data issues and their reconstruction, section 3 presents univariate statistical analysis, section 4 aims at estimating the bivariate VARs. Concluding remarks are relegated in the final section.

## **2. DATA ON THE NON-OBSERVED ITALIAN ECONOMY. ISSUES AND RECONSTRUCTION**

The source of the data both for the non-observed and for the regular sectors in Italy is Istat. The hidden production represents, according to SNA definition (SNA, 1993), the area of (legal) production activities that are not directly observed due to reasons of economic nature (deliberate desire to avoid taxes and/or to avoid observing the law provisions concerning the labour market) and/or statistical nature (*e.g.* due to the failure to fill out the administrative

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<sup>1</sup> The number of the full time equivalent units are equal to the number of jobs corresponding to full time. The total of full time equivalent units is obtained by the sum of (primary and secondary) full-time jobs and part-time jobs transformed into full-time units (Eurostat, 1995).



forms or statistics questionnaires). Istat claims that non-observed does not mean non-measured (Calzaroni, 2000; Baldassarini and Pascarella, 2003). Briefly, the method used for measuring black economy consists in i) the use of sources and survey techniques that make possible to measure the weight of unregistered work (this is achieved primarily by using labour status particulars declared by respondents in the household surveys: it is assumed that individuals have less reasons than enterprises to conceal the nature of their work); ii) the correction of the under-reporting of income by the enterprises through adjustments of the per capita production and value added values declared by the small production units (fewer than 20 employees) and iii) the checks for the consistency of the economic aggregates through the balancing of the resources and uses made at the level of each industry. As a result, Istat publishes annual estimates of the irregular<sup>2</sup> input of labour ( $l_j$ ) and it is able to quantify the underground value added and its weight on GDP. Needless to say, although the method is internationally recognised to be a very good one, it is not immune from concerns and problems. For instance, even if it is reasonable to assume that individuals have less reasons than enterprises to conceal the nature of their work, Boeri and Garibaldi (BG, 2002) point out that if employees cooperate in shadow activities they may decide not to declare to be working. As reported in their paper, a joint Istat-Fondazione Curella survey reports that about 25% of the black economy is wrongly assigned to the inactive status by the labour force survey. Also, some individuals who indicate to their interviewer that they are self-employed may actually be labouring in the underground economy. A study of the US internal revenue service<sup>3</sup> found that 47% of the workers who were classified as independent contractors did not report any taxable income. Another matter of caution is due to the hypothesis of equal productivity between workers, since as reported in BG irregular workers should have a lower productivity than their regular counterparts.

Istat yearly data on the non-observed sector computed according to the new version of the European System Account (ESA95) are available only for the period 1992-2001. However there is a previous release, computed according to the old version of the European System Account (ESA79), covering the period 1980-1997. By far the most important change between these two versions is due to multiple and occasional jobs, because the earlier system considered irregular all of them, while improvements in the methodology allows now to discriminate legitimate jobs in these specific categories of employment as well. However, the

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<sup>2</sup> Even if Istat knows (and surveys) only regular firms, from households' answers it can detect irregular workers engaged both for regular and for irregular firms. Also, Istat tries to take into account non-resident undocumented foreigner workers, which can not be observed directly by the usual sources used to uncover other kinds of black economy.

<sup>3</sup> Budget of the United States Government, 1984, p. 5-120.

differences between ESA79 and ESA95 irregular labour input (in full time equivalent units) impact only on the composition, but do not affect the total input of labour (Calzaroni, 2000). A similar logic holds for the GDP because, given the total GDP and the new information available, Istat is now able to re-allocate part of the ESA79 irregular GDP (the so-called underground area for statistical reasons) to the regular GDP. Also, Istat has released GDP according to ESA95 since 1980, thus 1980-2001 consistent estimates of total GDP and of the total input of labour ( $l_t$ ) are available. In other words, limiting the analysis to the non observed economy<sup>4</sup>, all that the new ESA implies is a zero-sum game reshuffling part of the FTE, and of the GDP, from the irregular to the regular side of the economic system. Therefore it seems to be useful to write:

$$\text{regular GDP} \equiv Y_r = (1 - \frac{l_i}{l_t}) * \text{GDP}$$

$$\text{irregular GDP} \equiv Y_i = \text{GDP} - Y_r$$

where all the variables, but  $l_i$ , are ESA95 consistent and available from 1980. Comparing the two versions of the regular GDP (namely,  $Y_{r79}$  and  $Y_{r95}$  according to the year of the ESA to which they refer) instead of the labour input data is a fundamental device, because the strong differences in the dynamics of the share of the underground FTE units ( $\frac{l_i}{l_t}$ ) between the old ( $l_{i79}$ ) and the new ( $l_{i95}$ ) series (quasi)annihilate when we refer to  $Y_r$ . This is so because, apart from a constant, the only difference between  $Y_{r79}$  and  $Y_{r95}$  is due to the difference ( $l_{i79} - l_{i95}$ ), whose impact on  $Y_{r79}$  and  $Y_{r95}$  is a very little, hopefully irrelevant, percentage. A visual idea of what I am speaking about can be drawn by looking at the table 1 and at the figures 1-2 reported in appendix 1. Clearly, within the present framework the shares of the irregular activities as percentage of GDP and of the black FTE as percentage of total FTE are equal by construction<sup>5</sup>. Summing up, despite computational innovations it seems plausible to use the common years shared by  $Y_{r79}$  and  $Y_{r95}$  to generate a new

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<sup>4</sup> If the difference between ESA79 and ESA95 GDP was only due to the different composition of the total number of workers, then these two versions of GDP would be equal. However, the ESA95 GDP is different from the ESA79 one for other account innovations. This is due to, eg., the more widespread reference to the “accrual accountability”, the different treatment of software production etc.

<sup>5</sup> Should also be clear that the present method generates real data for the shadow economy, while Istat offers nominal data. Of course, by construction, the share of non-observed economy on total GDP is the same both using real and nominal values.

1980-2001 series to be used in the empirical analysis of the next sections<sup>6</sup>. This can simply be done by regressing the 1995 version of the regular GDP (as above defined) on a constant and on the 1979 version of the regular GDP. Results show that the forecast ability is very high (see Appendix 1, table 2) and we can be sufficiently confident in using the reconstructed long series (see Appendix 1, figures 3-3a). Furthermore, the dynamics of the new series are somewhat confirmed by other estimates (Ministero del Lavoro, 1987; Bovi, 1999; Schneider and Enste, 2000), which report a strong growth of the black sector in the 1980s, followed by a slowdown in the more recent dynamics. Lastly, it is consistent both with the positive correlation between unemployment and shadow employment, and with the reversion of the jobless economic growth started in the mid-1990s (Boeri and Garibaldi, 2002; Bertola and Garibaldi, 2003).

### **3. UNIVARIATE ANALYSIS**

The first necessary step before validly estimating and using a VAR model is the univariate analysis of the stochastic properties of the series involved. The attention devoted to this topic is well deserved for several reasons. First, in contrast to stationary or trend stationary time series, models with a stochastic trend have time dependent variances that go to infinity with time, thus they are persistent in the sense that shocks have permanent effects on the values of the process. Second, when a series is used in regressions with other variables the interpretation of the regression results can depend on whether the variables involved are trend (TS) or difference stationary (DS). This phenomenon is related to the “nonsense” and “spurious” regression literature due to Yule (1926) and Granger and Newbold (1974).

It is also well known that unit root tests are based on asymptotic critical values. One expects in finite samples that the use of asymptotic critical values will result in over-rejection, and twenty-two (1980-2001) observations are definitively a finite sample. I address this potential problem by studying the properties of the total real GDP, which is available from 1960 to 2003 (drawn from the OECD online data base). The logic is straightforward. On the one hand, because of Istat reconstructions, the GDP series contains the regular and the irregular components even for the period 1960-1979 (Istat released only the total GDP for this period). On the other hand, once I know the statistical properties of the total GDP, I can use the algebra of integrated variables to infer the properties

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<sup>6</sup> This time series is available on request from author.

of the GDP components. Granger and Hallmann (1991) show that for a pair of independent variables holds (using a widespread notation)<sup>7</sup>:

$$I(0) + I(0) = I(0);$$

$$I(1) + I(0) = I(1);$$

$$I(1) + I(1) = I(1).$$

If the two series are cointegrated, then  $I(d) + I(d) = I(d-1)$ , where  $d$  is the order of integration.

Even forty-four years may prove insufficient for valid asymptotic inferences so, to assess the robustness of the results I perform three unit root tests. The first (NP) was worked out by Ng and Perron (2001). It yields both substantial power gains and a lower size distortions over the standard unit root tests, maintaining the null of unit root. NP offer four test statistics based on the GLS detrended data  $y_t^d$ . Altogether these statistics are enhanced versions of Phillips-Perron  $Z_\alpha$  and  $Z_t$  statistics (1988), the Bargava (1986)  $R_1$  statistic, and the Elliot *et al.* Point Optimal statistic (1996):

$$MZ_\alpha = (T^{-1} \sum y_t^d)^2 / 2\kappa$$

$$MSB = (\kappa / f_0)^{1/2}$$

$$MZ_t = MZ_\alpha \times MSB$$

$$MPT = \frac{\sum y_t^d}{c} \frac{\sum y_t^d}{\kappa - cT} / f_0 \quad (\text{if exogenous} = \text{constant})$$

$$MPT = \frac{\sum y_t^d}{c} \frac{\sum y_t^d}{\kappa + (1-c)T} / f_0 \quad (\text{if exogenous} = \text{constant, trend})$$

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<sup>7</sup> The result is more general than here reported because it refers to any linear combination of the variables.

where  $\kappa = \sum_{t=2}^T (y_{t-1}^d)^2 / T^2$  and  $f_0$  is an estimate of the residual spectral density at the zero frequency<sup>8</sup>. The choice of the autoregressive truncation lag,  $p$ , is critical for correct calculation of  $f_0$ . Here  $p$  is chosen using the modified AIC suggested by Ng and Perron (2001).

The second is the KPSS test (Kwiatkowski *et al.* (1992)), which can be thought as complementing the NP one because it tests the null hypothesis that real GDP is a TS stochastic process. Suppose the NP test fails to reject the unit root null because of low power. The KPSS test which has (trend) stationarity as the null should indicate the data have no unit roots. On the other hand, if the KPSS test rejects the trend stationarity null, then we have stronger evidence for unit root persistence. That is, consistent results from NP and KPSS tests yield more persuasive evidence on data persistence, while conflicting results indicate uncertainty associated with the interpretation of the individual test outcomes. The KPSS test is based upon the residuals from the OLS regression of  $y_t$  on the exogenous variables  $x_t$ :

$$y_t = x_t' \delta + u_t$$

The LM statistic is defined as:

$$LM = \sum_t S(t)^2 / (T^2 f_0),$$

where  $f_0$  is an estimator<sup>9</sup> of the residual spectrum at frequency zero and where  $S(t)$  is a cumulative residual function:

$$S(t) = \sum_{r=1}^t \hat{u}_r$$

based on the residuals  $\hat{u} = y - x' \hat{\delta}(0)$ . I maintain the same lag length selection criterion already used in the NP test.

Finally, I rely on a multivariate method as well. Hansen (1995) shows that incorporating information from related time series has the potential to enormously increase the power of unit root tests (see also Elliott and Jansson, 2003). Basically, the test is a multivariate version of the ADF test (that is why it

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<sup>8</sup> The frequency zero spectrum method used is the AR-GLS detrended.

<sup>9</sup> The frequency zero spectrum method used is the Kernel-Bartlett sum-of-covariances.

is called Covariate Augmented Dickey Fuller, CADF, test) and it exploits the information in related time series to improve power of stationarity tests and dominate their univariate counterpart whenever the correlation between the covariates and the dependent variable is non zero. When the zero frequency correlation is zero, these tests coincide with the univariate tests. As additional variable I select the labour input, a natural choice given the supply-side approach followed by Istat to estimate the shadow economy. Specifically, I regress the growth rate of GDP on a constant, time, the lag log-level of GDP, one lag of the growth rate of GDP, and one lag of the log-level of total employment<sup>10</sup> in full time equivalent units. I then perform an F-test for the null hypothesis that the coefficient on the lag level of log GDP and the coefficient on time are jointly zero. This amounts to a test of the null hypothesis that the GDP is difference stationary, against the alternative that it is stationary about a linear trend.

Results reported in Appendix 2 (table 3) show univocal evidence that the level of Italian real GDP follows an I(1) process around a deterministic trend. NP and CADF tests fail to reject the null of unit root, KPSS rejects the null of stationarity. It holds when the tests are applied both to the logarithmic and to the natural level of the GDP. According to the above reported algebra, one can expect that  $Y_r$  and  $Y_i$  be DS or TS, but they should not be cointegrated because otherwise the GDP would be a stationary process. Actually, a unit root in GDP could be validly consistent with the cointegrated and I(2) nature of both  $Y_r$  and  $Y_i$ . I rule out this event because it would imply an accelerating equilibrium rate of growth for both the GDP components. In fact, there are rare applications of cointegrated VAR model for I(2) real data, and usually this choice is based on economic arguments (Juselius, 2004). Furthermore, the VARs estimated in the next sections would be unstable. Finally, tentative applications of the NP and KPSS tests directly to  $Y_r$  and to  $Y_i$  show<sup>11</sup> that they could be DS or TS, but should not have a double root. Again, in the case of poor power tests it is always true that failure to reject a null hypothesis does not mean we can reject the alternative, so comparing NP and KPSS results is particularly relevant in the present context. To the extent  $Y_r$  and  $Y_i$  are not cointegrated, they do not share a common stochastic trend either, as shown by Stock and Watson (1988).

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<sup>10</sup> KPSS and NP tests show that this variable is clearly TS. I do not report these tests, but they are available on request.

<sup>11</sup> I do not report these tests, but they are available on request.

#### 4. VECTOR AUTOREGRESSION ANALYSIS

The previous section concluded that the level of Italian real GDP is a DS process, and that we remain with only three possible outcomes for its components: i) both  $Y_r$  and  $Y_i$  are two (independent) DS processes, ii) and iii), alternatively, one is TS and the other is DS. They can not be cointegrated, neither both TS because these events contrast with the  $I(1)$  nature of the GDP. One way to carry on notwithstanding this “veil of ignorance” is to perform a battery of vector autoregression models according to the stochastic properties of the two components of the GDP. Through the analysis of the covariances, the VAR approach allows us to see if one market has a tendency to lead the other, if there are feedbacks between them, if there are contemporaneous movements, and how do impulses (shocks, innovations) transfer from one sector to another. The VAR approach (Sims, 1980) sidesteps the need for structural modelling by treating every endogenous variables in the system as a function of the lagged values of all the endogenous variables in the system. Consider the VAR(p) model

$$\Phi(L)y_t = \varepsilon_t$$

where  $\Phi(L) = I - \Phi_1L - \Phi_2L^2 - \dots - \Phi_pL^p$ .

A basic assumption in the above model is that the residual vector follows a multivariate white noise. Also, in order that the VAR-model is stationary, it is required that roots of  $|I - \Phi_1z - \Phi_2z^2 - \dots - \Phi_pz^p| = 0$  lie outside the unit circle. Provided that the stationary conditions hold we have the vector moving average representation of  $y_t$  as

$$y_t = \Phi^{-1}(L)\varepsilon_t = \varepsilon_t + \sum_{i=1}^{\infty} \psi_i \varepsilon_{t-i}$$

where  $\psi_i$  is an  $m \times m$  coefficient matrix. The  $\varepsilon_t$ 's represent shocks in the system.

Suppose we have a unit change in  $\varepsilon_t$  then its effect in  $y$   $s$  periods ahead is  $\frac{\delta y_{t+s}}{\delta \varepsilon_t} =$

$\psi_s$ .

Accordingly the interpretation of the  $\psi$  matrices is that they represent marginal effects, or dynamic multipliers, or the model's response to a unit shock (or innovation) at time point  $t$  in each of the variables. The response of  $y_i$  to a unit shock in  $y_j$  is given by the sequence, known as the impulse multiplier function,

$$\psi_{ij,1}, \psi_{ij,2}, \psi_{ij,3}, \dots,$$

where  $\psi_{ij,k}$  is the  $ij^{\text{th}}$  element of the matrix  $\psi_k$  ( $i, j = 1, \dots, m$ ). Generally an impulse response function traces the effect of a one-time shock to one of the innovations on current and future values of the endogenous variables. Otherwise stated, the impulse response functions traces out how the variables will deviate from the path predicted by the model if there is a forecast error with respect to a specific equation at time  $t$ . Unforeseen movements in  $y_j$  are referred to as shocks and the state of the economy at the time  $t+m$  as responses. However, unless the error covariance matrix  $E(\varepsilon_t \varepsilon_t')$  is a diagonal matrix, the shocks will not occur independent from each other. The conventional practice in the VAR literature is to single out the individual effects by first orthogonalize the error covariance matrix, *e.g.* by Cholesky decomposition, such that the new residuals become contemporaneously uncorrelated with unit variances. Unfortunately orthogonalization is not unique in the sense that changing the order of variables in  $y$  changes the results. The economic theory may be used to solve the ordering issue. The approach I follow here is agnostic and it is based on trying the two possible orderings (because of the bivariate VAR) to see whether the resulting interpretations are consistent. Since in a bivariate model the Granger-causality implies that one variable must react to a shock of the other, within this framework I can address the causality issues as well.

The uncorrelatedness of the new residuals allows the error variance of the  $s$  step-ahead forecast of  $y_{it}$  to be decomposed into components accounted for by these shocks. Because the innovations have unit variances, the components of this error variance accounted for by innovations to  $y_j$  is given by

$$\sum_{k=0}^s \psi_{ij,k}^{*2}$$

where  $\psi_i^*$  is the orthogonalised version of  $\psi_i$ . Comparing this to the sum of innovation responses we get a relative measure how important variable  $y_j$  innovations are in the explaining the variation in variable  $i$  at different step-ahead forecasts, *i.e.*,

$$R_{ij,s}^2 = 100 \frac{\sum_{k=0}^{s-1} \psi_{ij,k}^{*2}}{\sum_{h=1}^m \sum_{k=0}^{s-1} \psi_{ih,k}^{*2}}$$



Thus, while impulse response functions traces the effects of a shock to one endogenous variable on to the other variables in the VAR, variance decomposition separates the variation in an endogenous variable into the component shocks to the VAR. Clearly, even the variance decomposition results depend on the ordering when there is contemporaneous correlation between the residuals. Again, for the robustness of the findings I replicate the two possible orderings of the bivariate VAR.

Another useful and workable set of experiments within the present statistical-atheoretical context is the analysis of the generalised impulse response functions. Pesaran and Shin (1998) have suggested a theoretically neutral way of deriving impulse responses that takes into account the information on the correlation of errors contained in the error covariance matrix. These authors construct an orthogonal set of innovations that does not depend on the VAR ordering. The generalized impulse responses from an innovation to the  $j^{\text{th}}$  variable are derived by applying a variable specific Cholesky factor computed with the  $j^{\text{th}}$  variable at the top of the Cholesky ordering. It should be noted that the generalised response profiles derived in this way are not conveying information about economic causation among the variables. The exercise can be thought of as tracing out how the observation of a forecast error in one equation of the system would lead to revisions in the forecast path of all model variables.

Summing up, according to the hypothesised statistical properties of the time series and to the findings of the third section, I perform three VAR models<sup>12</sup>:

Model 1  $Y_r \sim DS; Y_r \sim TS;$   
Model 2  $Y_r \sim TS; Y_r \sim DS;$   
Model 3  $Y_r \sim DS; Y_r \sim DS.$

The analyses of VAR residuals reported in the appendix 2 (tables 4-6) suggest that the VARs seem to provide a fair description of the information in the data. Evidence satisfy both normality and the white noise assumption. The following figures (Appendix 3) plot the relative mean estimates of the (Cholesky and Generalised) impulse response functions and show the variance decomposition outcomes. The pure shape of impulse functions is not fully informative of whether a detected reaction path is also meaningful in a statistical sense. Thus I also display the upper and lower limits of a 95% Monte Carlo band. Clearly, if these bands contain the zero line one can conclude that there is evidence of no reaction. All these models have the same exogenous variables, namely a

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<sup>12</sup> Both the variables are logged because was not possible to obtain multivariate normal residuals using natural values.

constant and a linear time counter, but (unreported) sensitivity analyses conduct adding a quadratic trend do not substantially change the stylised facts that emerge. These latter may be summarised in the following statements:

- the Italian real GDP seems to be composed by two orthogonal components, one regular, one irregular. In particular,
- the non-observed economy shows neither Granger-causality, nor co-movements with regard to the regular activities;
- a less univocal evidence shows that the observed economy might react to shocks hitting the shadow economy.

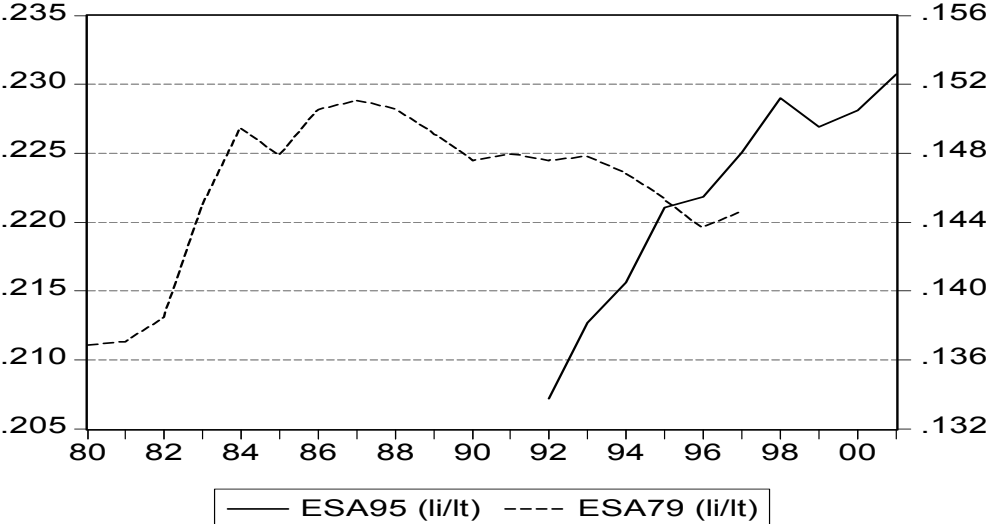
## CONCLUDING REMARKS

Starting from existing official estimates of the Italian non-observed economy, I have constructed and analysed a time series for the Italian shadow economy throughout the period 1980-2001. Several univariate unit root tests suggest that the regular and the “dark” side of the real GDP should not be cointegrated. In turn, this implies that they do not share a common stochastic trend. Then, according to the DS and/or TS nature of the GDP components, a battery of unrestricted VARs is performed to see whether the two sides of the economy are linked somehow. A visual inspection of the plots of impulse response functions and of the innovation accounting reveals that, no matter which model one prefers, the non-observed economy follows an univariate process. The results are not so univocal as regard to the regular GDP, which to some extent seems to be affected by shocks hitting the dark sector. Sensitivity analyses based on different deterministic variables confirm the outcomes, and it is worth recalling that statistical experiments have stronger ability in negating than in supporting the occurrence of an event. Of course, I can not exclude that the outcomes are biased because of measurement errors, such as black workers with a wrongly assigned labour status or different productivities between workers.

In this paper my target is to establish stylised facts rather than to explain them. However, I am tempted to speculate in order to offer some tentative comment. For instance, if one think about the shadow employment as a buffer pool, the univariate nature of the underground activities may be explained by the presence of alternative “regular” tools for reacting to negative shocks, without increasing the number of hidden workers. As a matter of fact, in the decades under scrutiny early retirements (prepensionamenti), the special wage supplementation fund (Cassa Integrazione Guadagni Straordinaria), the unduly increase of public sector employment, and the quasi-dependent (but formally self-employed) “collaborazione coordinata e continuativa” employment relationship might have been used for this purpose. The evidence that some percentage of the regular GDP variance might be due to shocks striking the shadow sector may find an explanation in the hiring subsidies and, especially, in the reiterate tax and foreign workers amnesties (“regularizations”), which impinge on the underground market before than, if any, on the regular one. Deeper and interesting analyses, *e.g.* to account for the potential informative content of variables such as the tax rate, are hampered by the scarcity of data and, at the moment, are relegated in the agenda.

**APPENDIX 1. Non-Observed Labour Input and Non-Observed GDP.**

**Figure 1. Old (ESA79, left scale) and New (ESA95) Non-observed Labour Input on Total (li/lt) Italy 1980-2001**



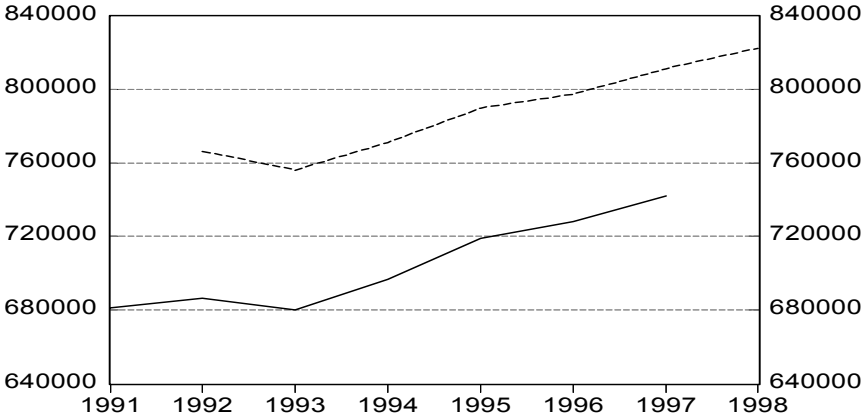
**Tab. 1 Regular GDP growth rates**

	$\Delta\%Yr79$	$\Delta\%Yr95$
1993	-0.918882	-1.380456
1994	2.469364	2.020472
1995	3.193336	2.431864
1996	1.279225	0.940338
1997	1.896374	1.744111

$\Delta\%Yr79$ =Italian real regular GDP (ESA79)

$\Delta\%Yr95$ =Italian real regular GDP (ESA95)

**Figure 2. Old (ESA79 ----) and New (ESA95 —) Italian real regular GDP.**



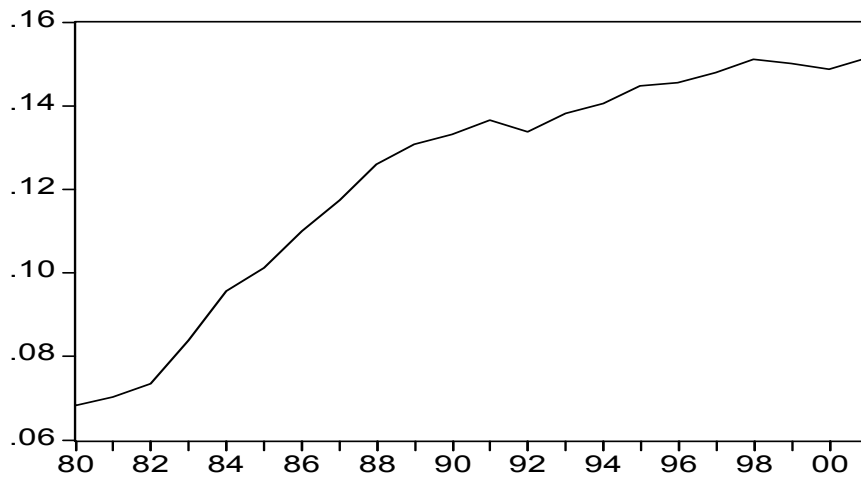
**Table 2 Forecast result**

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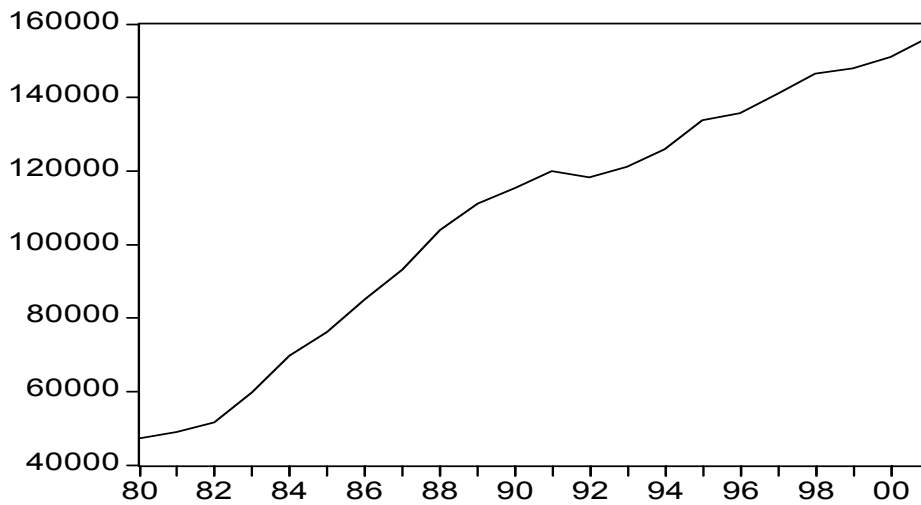
Forecast ability based on	
$y_{r95} = 185199 + 0.842 \cdot y_{r79}$	
Forecast sample: 1992 1997	
Root Mean Squared Error	1766.272
Mean Absolute Error	1516.073
Mean Absolute Percentage Error	0.195465
Theil Inequality Coefficient	0.001129
Bias Proportion	0.000000
Variance Proportion	0.002143
Covariance Proportion	0.997857

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**Figure 3. Italian Shadow Economy (on total GDP)  
1980-2001**



**Figure 3a. Italian Shadow Economy (millions of Euro)  
1980-2001**



## APPENDIX 2. Univariate and VAR Residual Analyses

Table 3. Unit root tests on Italian real GDP (annual data 1960-2003)

		MZa	MZt	MSB	MPT	KPSS*	CADF**
<b>Test statistics</b>	<b>GDP</b>	<b>-6.07492</b>	<b>-1.52662</b>	<b>0.25130</b>	<b>14.8071</b>	<b>0.211930</b>	<b>1.48</b>
	<b>D(GDP)</b>	<b>-21.2950</b>	<b>-3.23266</b>	<b>0.15180</b>	<b>4.46244</b>	<b>0.061826</b>	<b>----</b>
	<b>Log(GDP)</b>	<b>0.17475</b>	<b>0.10972</b>	<b>0.62787</b>	<b>87.9691</b>	<b>0.217302</b>	<b>2.66</b>
	<b>Dlog(GDP)</b>	<b>-79.1260</b>	<b>-6.28986</b>	<b>0.07949</b>	<b>1.15184</b>	<b>0.118034</b>	<b>----</b>
<b>Critical values</b>	1%	-23.8000	-3.42000	0.14300	4.03000	0.216000	5.16
	5%	-17.3000	-2.91000	0.16800	5.48000	0.146000	3.22
	10%	-14.2000	-2.62000	0.18500	6.67000	0.119000	2.42

Lag length criterion: Modified AIC; constant and trend included. \*H0: TS process; \*\*F-test for H0: DS vs TS process.

Table 4. Diagnostic tests on the VAR residuals. Model 1:  $Y_r \sim DS$ ;  $Y_i \sim TS$ . Two lags. Sample 1980-2001.

Single equation tests				
Portmanteau 3 lags $Y_i = 1.515$ $D(Y_r) = 2.462$	AR 1-2F(2, 11) $Y_i = 0.277$ [0.76] $D(Y_r) = 1.236$ [0.33]	Normality $\chi^2$ $Y_i = 2.7703$ [0.25] $D(Y_r) = 2.354$ [0.31]	ARCH 1 F(1, 11) $Y_i$ $= 0.671$ [0.43] $D(Y_r) = 0.740$ [0.41]	$\chi^2$ F(8, 4) $Y_i = 0.30$ [0.93] $D(Y_r) = 0.478$ [0.83]
Recursive residuals (Cusum and Cusum square) show no signs of instability				
Multivariate tests				
Vector portmanteau 3 lags $=$ 7.7407 [0.1016]	Vector AR 1-2 F(8, 16) $=$ 0.9936 [0.4766]	Vector normality $\chi^2$ (4) = 6.9061 [0.1409]	Vector $\chi^2$ F(24, 6) $=$ 0.23266 [0.9956]	

$D(x)$ =first difference of variable x; endogenous variables in logs; constant and trend included; degrees of freedom of the tests in parentheses; p-values in squared brackets;

Table 5. Diagnostic tests on the VAR residuals. Model 2:  $Y_r \sim TS$ ;  $Y_i \sim DS$ . One lag. Sample 1980-2001.

Single equation tests				
Portmanteau 3 lags $D(Y_i) = 2.326$ $Y_r = 1.407$	AR 1-2F(2, 14) $D(Y_i) = 3.65$ [0.053] $Y_r = 0.932$ [0.42]	Normality $\chi^2$ $D(Y_i) = 0.02$ [0.99] $Y_r = 2.05$ [0.36]	ARCH 1 F(1, 14) $D(Y_i) = 0.104$ [0.75] $Y_r = 1.2219$ [0.2876]	$\chi^2$ F(4, 11) $D(Y_i) = 1.69$ [0.22] $Y_r = 1.3788$ [0.3035]
Recursive residuals (Cusum and Cusum square) show no signs of instability				
Multivariate tests				
Vect. Portm. 3 lags $=$ 7.359 [0.49]	Vector AR 1-2 F(8, 22) $=$ 1.201 [0.3431]	Vect. normality $\chi^2$ (4) $=$ 1.966 [0.7420]	Vect. $\chi^2$ F(12, 24) $=$ 1.4166 [0.2253]	Vect. $X_i * X_j$ F(15, 22) $=$ 1.0635 [0.4370]

See legend under table 4.

Table 6. Diagnostic tests on the VAR residuals. Model 3:  $Y_r \sim DS$ ;  $Y_i \sim DS$ . One lag. Sample 1980-2001.

Single equation tests				
Portmanteau 3 lags $D(Y_i) = 0.76224$ $D(Y_r) = 2.4355$	AR 1-2F(2, 14) $D(Y_i) = 0.392$ [0.6830] $D(Y_r) = 1.217$ [0.3257]	Normality $\chi^2$ $D(Y_i) = 0.37$ [0.83] $D(Y_r) = 2.29$ [0.32]	ARCH 1 F(1, 14) $D(Y_i) = 0.005$ [0.94] $D(Y_r) = 0.50$ [0.49]	$\chi^2$ F(4, 11) $D(Y_i) = 0.57$ [0.68] $D(Y_r) = 1.68$ [0.22]
Recursive residuals (Cusum and Cusum square) show no signs of instability				
Multivariate tests				
Vect. Portm. 3 lags $=$ 6.0834 [0.64]	Vector AR 1-2 F(8, 22) $=$ 1.0879 [0.4074]	Vect. normality $\chi^2$ (4) $=$ 2.5622 [0.6335]	Vect. $\chi^2$ F(12, 24) $=$ 0.91443 [0.5475]	Vect. $X_i * X_j$ F(15, 22) $=$ 0.92194 [0.5552]

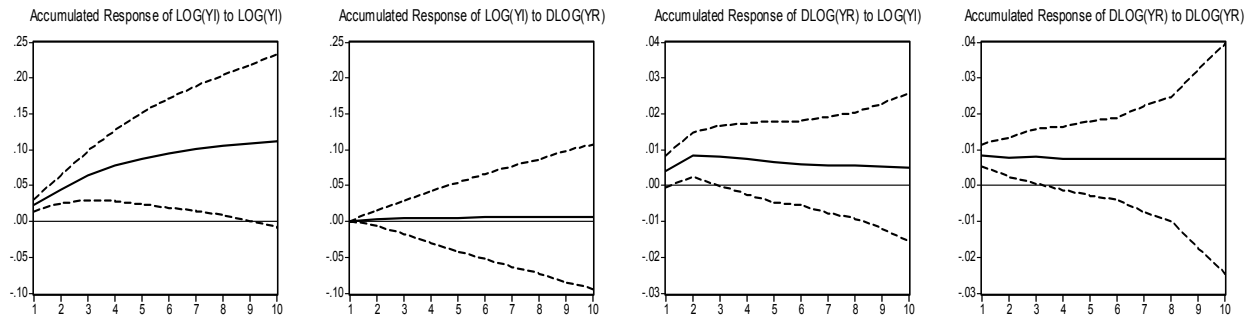
See legend under table 4.

### APPENDIX 3. Impulse Response and Innovation Accounting Analysis

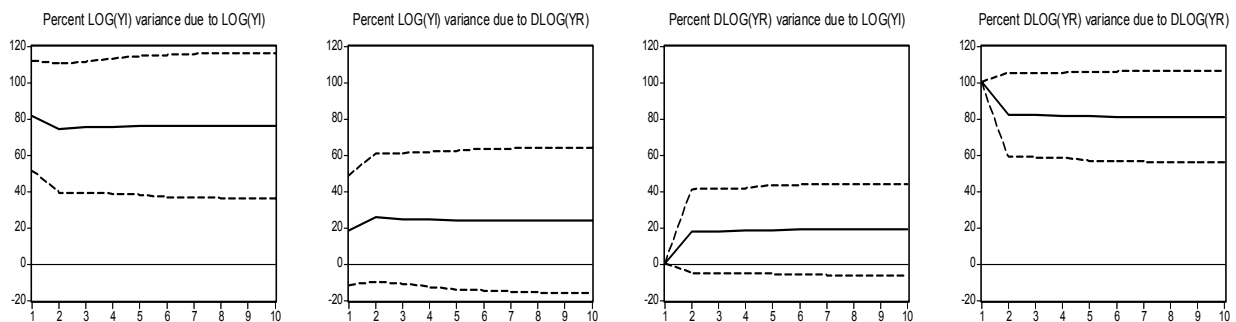
In all the models i) there are a constant and a linear trend; ii) the  $\pm 2$  S.E bands are drawn from 1000 Monte Carlo replications; iii) the Cholesky ordering for the relative impulse functions and for the variance decomposition analysis is  $Y_r - Y_i \Rightarrow Y_i - Y_r$ .

Model 1.  $Y_r \sim DS$ ;  $Y_i \sim TS$ .

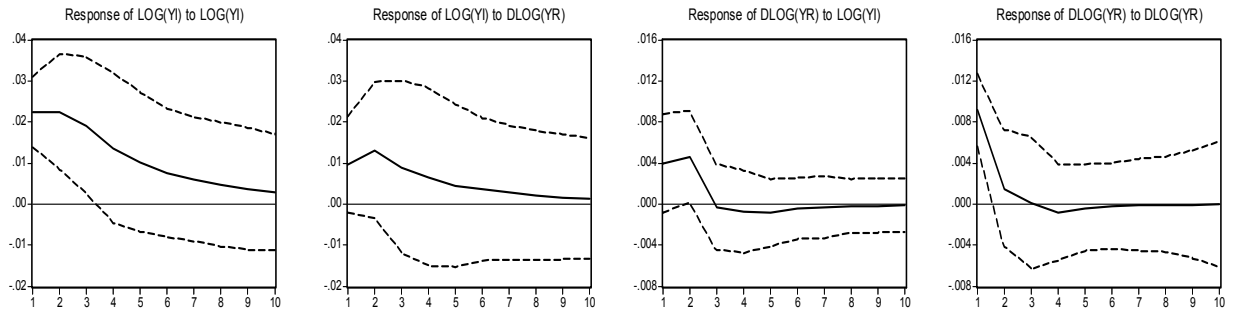
Accumulated Response to Cholesky One S.D. Innovations  $\pm 2$  S.E.



Variance Decomposition  $\pm 2$  S.E.

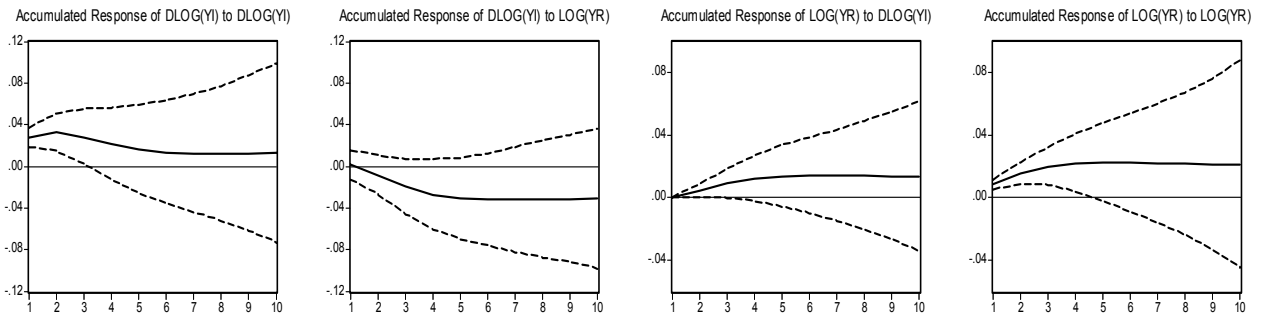


Response to Generalized One S.D. Innovations  $\pm 2$  S.E.

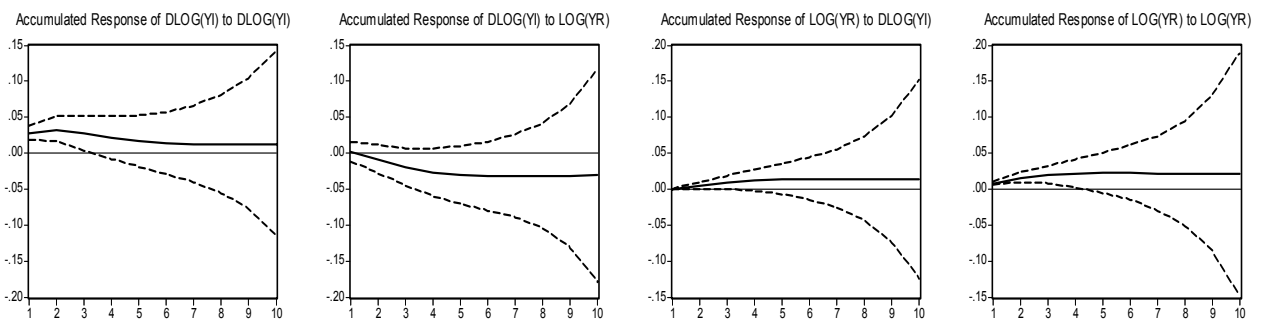


Model 2.  $Y_r \sim TS; Y_i \sim DS.$

Accumulated Response to Cholesky One S.D. Innovations  $\pm 2$  S.E.

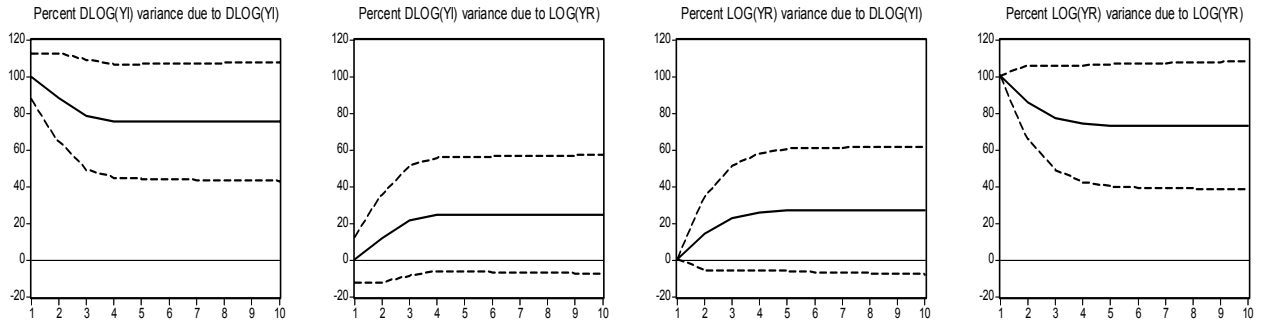


Accumulated Response to Cholesky One S.D. Innovations  $\pm 2$  S.E.

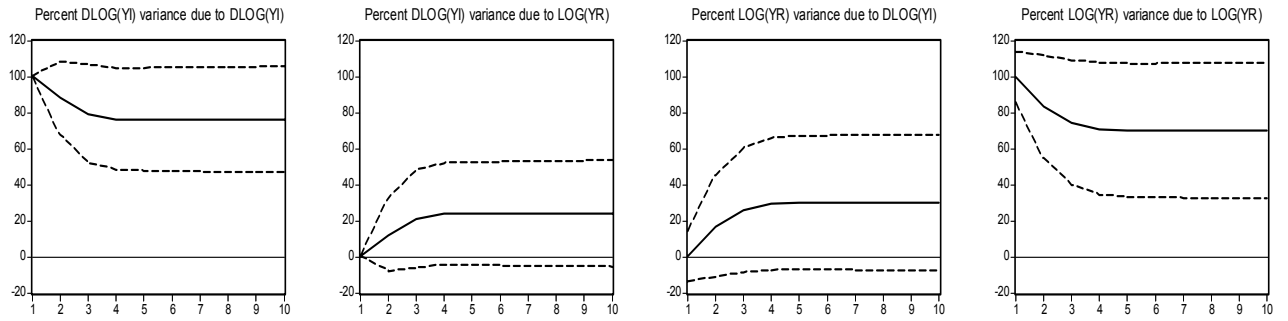




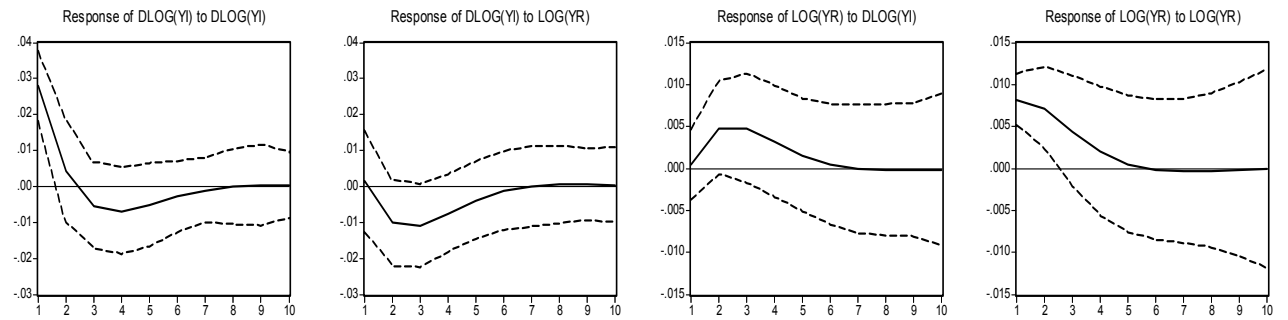
Variance Decomposition  $\pm 2$  S.E.



Variance Decomposition  $\pm 2$  S.E.

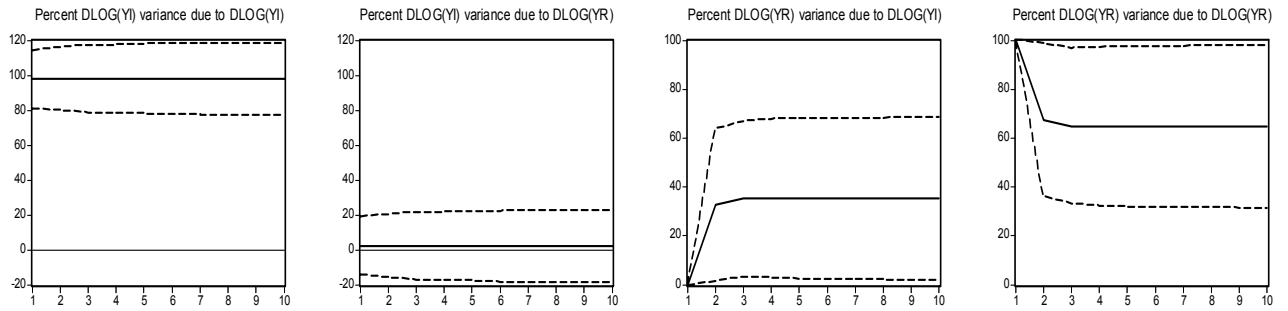


Response to Generalized One S.D. Innovations  $\pm 2$  S.E.

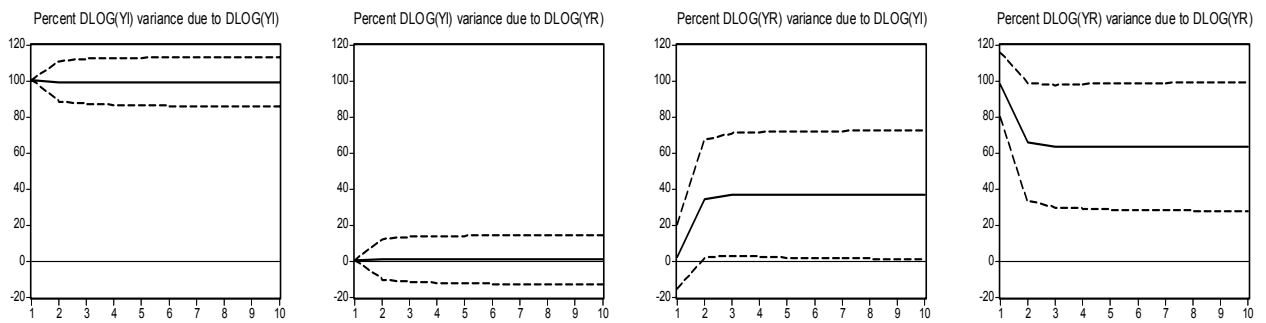


### Model 3. $Y_r \sim DS; Y_i \sim DS$

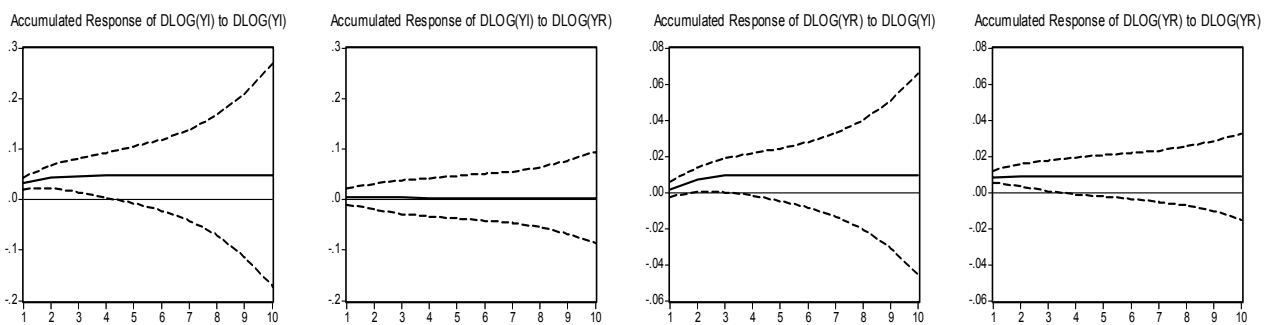
Variance Decomposition  $\pm 2$  S.E.



Variance Decomposition  $\pm 2$  S.E.



Accumulated Response to Generalized One S.D. Innovations  $\pm 2$  S.E.



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