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Statistical Criteria to Manage Non-respondents' *Intensive Follow Up* in Surveys Repeated along Time

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Abstract

When carrying out statistical surveys, detecting non-respondent units to undergo Intensive Follow Up (IFU) to disseminate quality estimates represents an important though under estimated aspect that is often not deeply studied. We suggest to use a generalised score function to quantify the risk related to the failed use of all the sample units for calculations together with techniques aiming at detecting IFU units through significance tests or methods – either parametric or not – based on acceptance thresholds. We also present an exercise related to a panel of industrial enterprises.

Keywords: Bias ratio, Pseudo bias, Response burden, Score function

1. Non response prevention and official statistics

In the field of official statistics, one of the main methodological issues is the well known trade/off between timeliness and accuracy. This applies especially to *short-term* statistics, that by definition are characterised by a very short delay between date of release and data reference period. When only a part of the assumed set of respondents is available for estimation, in addition to imputation or re-weighting (ISTAT, 2007), one should previously try to get responses by all those units that can be considered fundamental in order to produce good estimates. This applies both to census, cut/off and pure sample surveys.

On the other hand, there is a limit to the number of reminders that can be carried out, not only because of time and resource constraints, but also to contain response burden on enterprises involved in statistical surveys. The response burden assessment is becoming more and more relevant within the European Union from a strategic point of view (EUROSTAT, 2005*b*); the European Commission implicitly requests to limit it, thus influencing operative choices and forcing National Statistical Institutes to carefully consider how many and which non respondent units should be object of follow ups and reminders.

We indicate a generic *Follow Up* action as *FU* and an *Intensive Follow Up* action addressed to a subset of non respondent units as *IFU*. In this paper we will deal with the procedure to be used to identify this subset. Units belonging to this subset will be indicated as *IFUs*.

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In survey sampling, the most natural and recommended way to perform a *FU* process simply consists in choosing non-respondent random units to be re-contacted (Cochran, 1977, 365-367; Droesbeke, Fichet and Tassi, 1987, 181-182). More in detail, the following actions should be carried out:

a. determining the number of units to be re-contacted. This can be done according to the sample variance formula to ensure a minimum desired precision level of the sample, or on the basis of operational constraints and deadlines for provisional data publication. A further fundamental step is the detection of the lowest number of *IFUs* that can ensure a certain error level.
b. Choosing non-respondent units at random, according to preliminary inclusion probabilities. In this case, the two fundamental problems related to *IFU* – choice and identification of the units to be re-contacted – are faced in two separate steps, while in an *IFU* context they are generally solved simultaneously, according to the preliminary definition of an individual *score function* (Hedlin, 2003; Philips, 2003), as it will be described in sections 2 and 3.

However, there are several practical cases when resorting to an *IFU* strategy can be an alternative to the previous random selection procedure and, sometimes, even necessary:

- The available sample is not obtained from a predefined sample design, but is for instance a natural sample from administrative data. In this case the concept of sample variance looses its meaning if randomness is referred to the *sample design*.
- The available sample has been selected in a deterministic way, and/or according to a superpopulation model, so that the final *MSE* will be evaluated according to the model and not to sample design.
- The survey is a census, or a *cut/off* sample survey, so that normally the identification of an *IFU* strategy is based on the need to maximise the weighted response rate.
- Finally, even when the sample derives from a specific sample design, one would still have the possibility to apply reminders for a specific subset of non respondents. In this case the probability of final inclusion will change and final *MSE* estimation will be based on a mixture of old and new inclusion probabilities, related respectively to units responding without re-contacting or as a consequence of an *IFU* action.

Even though this topic is assuming increasing relevance, especially in the framework of official business statistics, it recently did not attract much attention. Moreover, the main available theoretical proposals often refer to data editing problems rather than to intensive follow-up management. In the light of what we said, our main purposes are:

1) to resume into a general methodology theoretical criteria and best current practices, focusing on the definition of generalised "score function" to be calculated for each non-respondent unit.
2) To propose a general "score function", valid to estimate both level and change.
3) To evaluate and compare some criteria for identifying critical units that should be considered as *IFUs* according to their score function.
4) To compare the various criteria by carrying out an empirical experiment using real business survey data.

Points 1) and 2) are dealt with in sections 2 and 4, point 3) in section 3 and point 4) in section 5; summing up conclusions are drawn in section 6.

2. The score function approach in the univariate case

A theoretical sample s with size n is drawn from a population composed by N units. When estimates are needed, only an effective sample s_R including n_R respondent units is available. If $s_{\bar{R}}$ is the sub-sample including the $n_{\bar{R}} = (n - n_R)$ non respondents, the main purpose is the identification of a sub-sample $s_{\bar{R}*}$ including the $n_{\bar{R}*}$ *IFU* units (*IFUs*) to be re-contacted, with $n_{\bar{R}*} \leq (n - n_R)$. These units are fundamental to ensure sufficiently good estimates[2] of the unknown population mean - or the variation of this mean between reference period t and a previous time (t-1) - and should be object of *IFU* in case of non-response or *late* response.

The following problems should be faced: a) the definition of a score function based on observed data expressing the *statistical risk* related to the lack of data referred to a certain unit; b) the choice of a statistical criterion able to detect which of these individual scores are particularly high, thus leading to the identification of *IFUs*.

It is worth remarking that all the following considerations cannot be applied to outstanding units who are new to the survey or had provided non-responses during all previous survey occasions. A precautionary option consists in assigning them the highest *IFU* priority.

From now on we can also suppose that only one variable is observed (or is considered as relevant).

2.1 Estimate of level

In this case the population mean $\bar{y}$ is the main estimation object. If y_i is the y-value reported by the i^{th} unit, and $\hat{y}_i$ is its estimate obtained by imputation technique – as that currently used in the survey – the first step consists in defining the following conversion:

$$r_i = \left| z_{1i}\, y_i - z_{2i} \right| \tag{1}$$

where the new variables z_1 and z_2 must be determined. In particular, one can assume:

a) $z_{1i} = 1$ and $z_{2i} = 0$ (2)

so that (1) represents the simple absolute y-value related to the unit;

b) $z_{1i} = 1$ and $z_{2i} = \hat{y}_i$ (3)

so that (1) becomes the absolute difference between true and estimated y-value;

c) $z_{1i} = \hat{y}_i^{-1}$ and $z_{2i} = 0$ (4)

so that (1) becomes the absolute ratio between true and estimated y-value;

[2] For *quality* of estimates, see section 3.

d) $z_{1i} = y_i^{-1}$ and $z_{2i} = \hat{y}_i y_i^{-1}$ (5)

so that (1) becomes the absolute relative difference between true and estimated y-value.

According to case b), function r_i is similar to that proposed by Mckenzie (2003, 476). Moreover, according to case c), function r_i becomes similar to that proposed by Latouche and Berthelot (1992, 392), even though in that case z_{2i} was given by the y-value referred to a previous time (t-1). Position d) as proposed by Gismondi (2007) has the advantage, with respect to b), to deal with functions independent from measuring unit and individual magnitude and that can be summed up over different units.

The next step consists in multiplying function r by the individual sample weight w_i, a second factor γ_i and a third factor measuring the importance of the unit, based on the value of the variable, so that a first score function will be given by:

$$\Phi_{1i} = r_i \cdot w_i \cdot \gamma_i \cdot (MAX(y_i, z_{1i}, z_{2i}))^U \tag{6}$$

and resorting to the *MAX* function is coherent with options b) and c), while in case a) one could assume U=0. The second factor γ_i is normally assumed equal to 1, thus disappearing, but it turns out useful when one has to group more than one score functions for the same unit i: this applies to short-term surveys, when we need to average the single *monthly* score functions by taking into account the different importance of each month (i.e. the different magnitude of monthly estimates). An operational example of that will be discussed in section 5.

As also remarked by Hidiroglou and Berthelot (1986), the exponent U ($0 \le U \le 1$) provides a "*control of the importance*" associated with the data magnitude. This parameter is not very sensitive and the same value can be used for many variables of the survey; we commonly have U=0.5.

According to case a), function (6) is equivalent to that used by Pursey (2003), Chen and Xie (2004) and Succi and Cirianni (2005) when U=0, assuming that the y-value used to implement (1) can be obtained by a business register; this assumption is realistic if y is equal to turnover or business revenue in general. In particular, also weights w are constant under a simple random sample design, so that Φ_{1i} simply represents the original value y_i. The main difference between a), b) or c) is that in these last two cases a large unit is not necessarily characterised by a high score function.

In the third step the first score function (6) is converted into a second score function (see following equation) – where $q_{(\Phi_1)0,25}$, $q_{(\Phi_1)0,50}$ and $q_{(\Phi_1)0,75}$ are the first, the median and the third quartile of the score function (6) respectively:

$$\Phi_{2i} = \left(\frac{\Phi_{1i} - q_{(\Phi_1)0,50}}{q_{(\Phi_1)0,75} - q_{(\Phi_1)0,25}} \right). \tag{7}$$

By this conversion, the final score function will have a more uniform and symmetric distribution than function (6), whose form is strongly influenced by the original y distribution (Gismondi, 2000). This aspect will be considered again in section 4.

A further choice for z_1 and z_2 can be obtained assuming to estimate the effect of missing units on final level estimate. If the i^{th} unit is non respondent, it is possible to estimate its y-value and to carry on the average level sample estimate by including this estimate in the calculation. Then, the score function for the i^{th} unit is given by the absolute difference between the estimates obtained by using the true y_i value (first round brackets in (8)) and those estimated (second brackets); if the estimator of the population mean is given by $N^{-1}\sum_{i=1}^{n} y_i w_i$, the score will be given by:

$$N^{-1}\left|\left(\sum_{j\neq i\ j=1}^{n-1} y_j w_j + y_j w_j\right) - \left(\sum_{j\neq i\ j=1}^{n-1} y_j w_j + \hat{y}_j\right)\right| = N^{-1} w_j \left|y_j - \hat{y}_j\right| \tag{8}$$

where the last term is similar to that obtained from (1) in case b), and the sampling weight w is still included in the score function before conversion (6).

Things change if we assume that the i^{th} non-respondent unit is not estimated and is excluded from calculations. That happens, for instance, when no imputation procedure has been planned for the survey, or for particularly large and relevant units[3], whose values must be obtained directly.

If the sample design is based on inclusion probabilities π_i, the sample weight used to estimate the unknown mean is given by $w_i^{(n)} = \left(\pi_i^{(n)}\right)^{-1}$, where the estimate is based on n units. If the sample design is based on (n-1) units, we can assume[4] that $\pi_i^{(n)} = \alpha\, \pi_i^{(n-1)}$ for whatever n. Since $\sum_{i=1}^{N} \pi_i^{(n)} = n$, it follows immediately that $\alpha = n\,(n-1)^{-1}$, so that the following relation will hold:

$$\pi_i^{(n)} = n\,(n-1)^{-1}\,\pi_i^{(n-1)} \quad \rightarrow \quad w_i^{(n)} = (n-1)\,n^{-1}\,w_i^{(n-1)} . \tag{9}$$

The absolute difference between estimates based on n and (n-1) units – where (-i) is the estimate based on all units except the i^{th} unit – will be given by:

$$\left|\hat{\bar{y}}^{(n)} - \hat{\bar{y}}^{(n-1)}_{(-i)}\right| = N^{-1}\left|\sum_{j=1}^{n} y_j w_j^{(n)} - \sum_{j\neq i\ j=1}^{n-1} y_j w_j^{(n-1)}\right| = N^{-1}\left|\left(\sum_{j\neq i\ j=1}^{n-1} y_j w_j^{(n)} + y_i w_i^{(n)}\right) - \sum_{j\neq i\ j=1}^{n-1} y_j w_j^{(n-1)}\right| =$$

[3] Small units can be still relevant if they belong to very small strata.
[4] Simple random sampling and *PPS* designs comply with this rule.

$$= N^{-1}\left|(n-1)\,n^{-1}\left(\sum_{j\neq i\quad j=1}^{n-1} y_j\, w_j^{(n-1)} + y_i\, w_i^{(n-1)}\right) - \sum_{j\neq i\quad j=1}^{n-1} y_j\, w_j^{(n-1)}\right| = n^{-1}\left|(n-1)\,N^{-1}\, y_i\, w_i^{(n-1)} - \hat{\bar{y}}_{(-i)}^{(n-1)}\right| =$$

$$= n^{-1}\left|\hat{\bar{y}}_{(i)}^{(1)} - \hat{\bar{y}}_{(-i)}^{(n-1)}\right|. \tag{10}$$

The previous quantity is proportional to the absolute difference between the population mean estimates based, respectively, only on the i^{th} unit and all the remaining (n-1) units except the i^{th}. Under a simple random sample design, the previous estimates are but sample means based on 1 and (n-1) observations respectively. The previous equation can be related to (1) as follows:

e) $z_{1i} = N^{-1}\, w_i^{(n)}$ and $z_{2i} = n^{-1}\, \hat{\bar{y}}_{(-i)}^{(n-1)}$ (11)

In order to calculate scores, some additional aspects should be taken into account:

- score functions can be estimated with reference to actual time t only if they are calculated using an auxiliary x-variable; in a longitudinal survey, often the same y-variable we referred to on one ore more previous occasions (t-1) is used. If a unit is included in the survey for the first time, it could be excluded from (or included in) follow-up actions *a priori*, or its score can be estimated according to an auxiliary x-variable related to y.
- If the survey is a census, sample weights w in (6) disappear. In order to ensure the validity of (6), they are assumed all equal to one.
- If stratification is used, score functions (6) should be defined and estimated separately in each stratum. However, if all units are considered as a whole, and W_v is the relative weight of the v^{th} stratum, we can evaluate the new function $W_v\Phi_{1vi}$[5].
- A more general way, rather than converting c) or d), to deal with comparable score functions consists in dividing conversions (2) or (3) by the true overall mean $\bar{y}$, or its estimate $\hat{\bar{y}}^{(n)}$. Even though this adding factor does not have consequences on the choice of *IFUs*, it can be helpful whenever it is necessary to sum score functions referring to different domains, that could be expressed in *different units of measurement* (section 3).

Finally, given the vectors of observations **y** and the scores **Φ**, a general rule useful to identify *IFUs* consists in analysing the behaviour of a general conversion *f* such as:

$$f(\Phi_{2i}, y_i)\,. \tag{12}$$

[5] In a stratified random sampling context, units to be re-contacted could be distributed among strata according to the Neyman allocation rule.

Function f can be based only on Φ_i, only on y_i or, more generally, on both. Some alternative options are discussed in section 4. It should be noted that identification of *IFUs* is not *necessarily* based on the definition of a threshold for f – e.g. some f^* – even though this possibility is explicitly considered in section 3.2.

2.2 Estimate of change

The main purpose of most short-term business surveys is to estimate the change $\bar{y}_t / \bar{y}_{(t-1)}$, where t is a month or a quarter and (t-1) is a generic previous period – for instance, the base year when index numbers are calculated. The individual change will be given by $c_{ti} = c_i = y_{ti} / y_{(t-1)i}$. In this case, it is useful to apply a further conversion to the individual change, given by $c^*_{ti} = MAX(y_{ti} / y_{(t-1)i}, y_{(t-1)i} / y_{ti})$. This option derives from the need to assign units characterised both by a very high *and* a very low change high priority, even though the next step (13) can be applied indifferently to c or c^*. Of course, final relevance of each unit will be determined according to its c^* value and its magnitude, according to function (6).

Even though the logical frame remains similar to that shown in section 2.1, a relevant difference is that the score function – given the individual y-magnitude – should increase whenever a unit is characterised by very high or very low rates of change over time. On the other hand, only the univariate case will be considered in detail, since all considerations on the multivariate case (section 4) remain valid to estimate change as well.

We also assume that the same units are included in the sample with the same inclusion probabilities for (t-1) and t.

The firs step consists in defining a conversion similar to (1), but applied to c_i:

$$r_i = |z_{1i} c_i - z_{2i}| \tag{13}$$

where the new variables z_1 and z_2 must be determined. In particular, the options that seem useful to estimate change are:

a′) $z_{1i} = 1$ and $z_{2i} = 0$ (14)

so that (13) is nothing but the simple c-value related to the unit;

b′) $z_{1i} = 1$ and $z_{2i} = \hat{c}_i$ (15)

so that (13) becomes the difference between the true and estimated c-values.

All the previous steps (6) and (7) can be applied, with obvious modifications, to conversion (13), so that a score function Φ_{2i} can be calculated also in this case.

A further choice for z_1 and z_2 can be obtained by evaluating the effect of the missing availability of a unit on final change estimate. If the i^{th} unit is a non-respondent unit and is excluded from calculation, assumption (9) on inclusion probabilities still remains valid and the symbols introduced in section 2.1 keep their meaning, the absolute difference between the estimates of change between times t and (t-1) based on n and (n-1) units is:

$$\left|\left(\frac{\hat{y}_t^{(n)}}{\hat{y}_{(t-1)}^{(n)}}\right)-\left(\frac{\hat{y}_{t(-i)}^{(n-1)}}{\hat{y}_{(t-1)(-i)}^{(n-1)}}\right)\right|=\left|\left(\frac{\sum_{j=1}^{n} y_{tj}\, w_j^{(n)}}{\sum_{j=1}^{n} y_{(t-1)j}\, w_j^{(n)}}\right)-\left(\frac{\sum_{j\neq i\quad j=1}^{n-1} y_{tj}\, w_j^{(n-1)}}{\sum_{j\neq i\quad j=1}^{n-1} y_{(t-1)j}\, w_j^{(n-1)}}\right)\right|=n^{-1}\left(\frac{\hat{y}_{(t-1)(i)}^{(1)}}{\hat{y}_{(t-1)}^{(n)}}\right)\left|I_{t(i)}-I_{t(-i)}\right| \quad (16)$$

where I_t is an index of change between times t and (t-1) and:

$$I_{t(i)}=\frac{y_{ti}}{y_{(t-1)i}}\ ;\quad I_{t(-i)}=\frac{\sum_{j\neq i\quad j=1}^{n-1} y_{tj}\, w_j^{(n-1)}}{\sum_{j\neq i\quad j=1}^{n-1} y_{(t-1)j}\, w_j^{(n-1)}}\ ;\quad \hat{y}_{(t-1)(i)}^{(1)}=N^{-1}n\, y_{(t-1)i}\, w_i^{(n)}\ ;\quad \hat{y}_{(t-1)}^{(n)}=N^{-1}\sum_{j=1}^{n} y_{(t-1)j}\, w_j^{(n)}\ .$$

In this case, the score function is based on the absolute difference between the indexes of change calculated, respectively, on the only i^{th} unit and on the (n-1) units excluded the i^{th}, and is conceptually similar to (11) referred to levels. However, an additional factor – with respect to the only individual y-magnitude – affecting the score is the ratio between the level estimates referred to time (t-1) calculated, respectively, on the only i^{th} unit and on the all n units. If (t-1) is the base year of index numbers, this ratio is the relative weight of the i^{th} unit on the overall level in the base year.

Thus, to estimate change the individual score function depends both on: 1) the contribution given by the unit to the overall level estimate at time (t-1) and 2) the difference between the individual trend and the overall average trend evaluated on the remaining (n-1) units. A relation with (1) can be easily obtained - given that $\hat{y}_{(t-1)(i)}^{(1)}\Big/\hat{y}_{(t-1)}^{(n)}=g(\mathbf{y}_{(t-1)})$ - through the following equation:

c′) $z_{1ti}=n^{-1}g(\mathbf{y}_{(t-1)})\,y_{i(t-1)}^{-1}$ and $z_{2ti}=n^{-1}g(\mathbf{y}_{(t-1)})\,I_{t(-i)}$. (17)

3. Identification of units to be re-contacted

The number of reminders can be determined in different ways. First, one can choose a score function among those described in sections 2 and 3; then, scores must be ordered in an increasing way; *IFUs* will be thus given by those units occupying the first positions in the ranking. The problem is the definition of a rule to decide how many *first positions* must be considered. The identification of *IFUs* can be obtained:

1) according to operational constraints, such as the maximum number of units that can be effectively followed with particular care, given technical and human resources devoted to the survey.
2) By evaluating the relation existing between reduction of pseudo-bias (section 3.1) and number of follow-ups.
3) On the basis of some other statistical test different from 2) carried out on the individual score functions.

In case 1), once the number of reminders that can be managed given the operational constraints and deadlines for publication has been fixed *a priori* , the choice of units can be done using rules as described in section 3.2. However, in current practice - especially under non probabilistic sample designs, or in case of *cut-off* sample- it is quite common to re-contact all and only units that, added to those already available, ensure a given coverage level referred to one or more main variables observed in the survey.

In both cases 2) and 3) the use of a score is joined to the search of a threshold able to discriminate between units to be and not to be re-contacted. Two main criteria have been resumed in sections 3.1 and 3.2. In the following, we can assume that:

- all the available n observations are independent from each other;
- we can deal in each stratum with a sufficiently large number of units, so that estimator distributions can be approximated by a normal density;
- score functions have been preliminarily ordered in a decreasing way.

3.1 Evaluation of the bias ratio and the pseudo bias

In the frame of case 2) mentioned above, we usually consider a "test data set", which either derives from some previous periods of the survey or is an early batch of data in the current survey period. This dataset should contain all the units, including non-respondent units at current time.

The underlying idea is to test the significance of the difference between the y-estimate based on a complete data set of respondents and the data set not including a certain unit. A fundamental aspect to be clarified is the form assumed by function f defined in (12): while score functions as (7) are used in order to create a ranking of units according to their not increasing score level, y-values are those *effectively taken into account* to test significance. If $\hat{\bar{y}}$ is the benchmark reference for assessing precision of the estimate $\hat{\bar{y}}_{(-i)}$ not including the i^{th} unit, we can evaluate the *bias ratio* of the estimate: since the global error of this estimate is the sum of squared bias and sample variance, the bias ratio is given by the incidence of the former error component on the latter - provided that variance under square root depends on estimator and sample design used:

$$BR(\hat{\bar{y}}_{(-i)}) = \frac{\left|\hat{\bar{y}} - \hat{\bar{y}}_{(-i)}\right|}{\sqrt{Var(\hat{\bar{y}}_{(-i)})}} \,. \tag{18}$$

On the basis of (18), the selective choice of units to be re-contacted can be driven by the evaluation of how much bias one should accept. Starting from the non-respondent unit with the highest score, one by one all non-respondent units are assumed to be excluded from calculations and considered in order to evaluate $\hat{\bar{y}}_{(-i)}$. If sample estimates approximately follow a normal distribution, the bias ratio is approximately $N(0,1)$.

We can also define the *coverage probability*, that is the probability that the unknown mean is contained within a confidence interval derived from the standardised normal distribution Z. This probability is given by: $\Pr[-z_{1-\alpha/2} - BR(\hat{\bar{y}}_{(-i)}) < Z < z_{1-\alpha/2} - BR(\hat{\bar{y}}_{(-i)})]$ - where $z_{(1-\alpha/2)}$ is the percentile of the standardised normal cumulated distribution leaving on the right a probability equal to $\alpha/2$. Thus the coverage probability equals the nominal, desired confidence level, $(1-\alpha)$, only if the bias ratio is equal to zero.

However, according to Cicchitelli, Herzel and Montanari (1992, 65-66) and Särndal, Swensson and Wretman (1993, 163-165), we can consider that a bias ratio lower than 10% results into a loss of coverage probability lower than 1%, which is therefore entirely negligible if compared with other shortcomings of common variance estimates. The operational rule in (18) consists in ordering units by decreasing score, identifying all the units for which the bias ratio is higher than 10% as *IFUs* and stopping as soon as the first unit with bias ratio under 10% is found.

It should be underlined that the use of (18) can be strictly connected with a statistical test useful to estimate the distance between one unit and a group of units. If we consider a generic X variable measured on n units, each X_i-value is compared with the mean $\overline{X}_{(-i)}$ calculated on the remaining units excluded the i^{th}. At the first step, when n units are considered, the test is based on $T_{(n-2)} = (X_i - \overline{X}_{(-i)}) \Big/ \sqrt{S^2_{X(-i)}\, n/(n-1)}$, where $S^2_{X(-i)}$ is the X-variance calculated on the whole sample excluded the i^{th} unit and $T_{(n-2)}$ is the Student's t with $(n-2)$ degrees of freedom. In its original version, the procedure – based on a unilateral test since $X_i > \overline{X}_{(-i)}$ – stops if the unit with the highest score is not detected as critical, otherwise it is carried out again after recalculation both of sample mean and variance.

Given that, it is easy to verify that from (18) – under a simple random sampling without replacement design and assuming $Var(\hat{\bar{y}}_{(-i)}) = \hat{\sigma}^2_{(-i)}/(n-1)$, where $\hat{\sigma}^2_{(-i)}$ is an estimate of σ^2 obtained using all units except the i^{th} – we have: $BR(\hat{\bar{y}}_{(-i)}) = T_{(n-2)}/\sqrt{n}$, so that, apart from a constant term, the two tests are similar.

Since this 10% threshold could be conservative, other choices can be used, for instance the empirical (*pseudo*) bias:

$$EB(\hat{\bar{y}}_{(-i)}) = \frac{\left|\hat{\bar{y}} - \hat{\bar{y}}_{(-i)}\right|}{\hat{\bar{y}}} \tag{19}$$

that can be calculated on the basis of late data referred to some previous period of the survey. A similar choice was done by Latouche and Berthelot (1992), with the aim to find the lowest number of reminders for which the empirical bias registers a strong decrease. However, different thresholds for evaluation (19) can be used, so that critical values of the empirical bias could be also evaluated according to the methods described in sections 3.2 and 3.3. Test functions based on (18) or (19) could also be used when the number of units that can be re-contacted is *given* due to operational constraints (as in the previous case 1)), in order to evaluate how large is the bias gap due to the impossibility to re-contact all the necessary units.

3.2 Parametric tests based on thresholds

When it is not possible to use complete datasets in order to evaluate (18) or (19), or just in order to carry out additional comparative tests, one can consider a series of statistical procedures based on the simple idea of verifying if a given unit belongs or not to the same populations of the others. Commonly, similar tests are used for identifying outlier observations in sample survey frames.

Herein X is a generic variable, that could be obtained by a score function as (7) or the relative gain in pseudo bias reduction (19). In both cases we also assume that units have been preliminarily ordered according to their decreasing X-values.

When the form of the X distribution is unknown, a very general and simple tool is provided by the Chebyshev inequality. If μ_X and σ_X are the population mean and standard deviation of X – that can be estimated according to previous surveys or current available observed data – and $Z=(X-\mu_X)/\sigma_X$, we can consider a specific X-value as critical if

$$1-(1/Z^2) > Pr \tag{20}$$

where Pr is a given probability level. Since the test is bidirectional, when suspected X-values are higher than μ_X, the choice Pr=0.10 means that the highest values representing 5% of the empirical density distribution are to be considered as critical. The main limits of this criterion are: 1) it is much less powerful than others based on knowledge of X distribution; 2) it does not supply an *exact* probability that the test function is critical.

A second criterion is based on the standardised normal distribution and on the assumption that Z-values are distributed in an approximately normal way. In this case, if both μ_X and σ_X are estimated using the whole available sample (*including* potential critical units), we can consider a specific X-value as critical if:

$$Z > z_{(1-\alpha)} \tag{21}$$

where $z_{(1-\alpha)}$ is the percentile of the standardised normal cumulated distribution corresponding to a probability equal to α, using an unidirectional test. When n<100 a better approximation can be achieved by using the Student's t distribution.

Test (21) and test (20) - when mean and variance are estimated using current sample data – should be carried out one unit at a time: when the first unit is identified as *IFU*, mean and variances should be recalculated, until there are no critical units left. The procedure is stopped immediately if the unit with the highest score is not detected as critical.

Even though test (21) is more precise and more powerful than test (20), its use depends on assumption of normality for X; moreover, Shiffler (1988) remarked that it can lead to wrong conclusions because the maximum limit for Z is $(n-1)/\sqrt{n}$, so that in the presence of a lower number of sample units it will be easier to identify one unit as critical, because the highest value that Z could reach will be lower.

A further test connected with Student's t is the *Extreme Studentized Deviate* test. It was originally proposed by Grubbs (1969) and in this context it will be based on:

$$(X_{Max} - \overline{X})/S_X \tag{22}$$

where X_{Max} is the highest X-value among the n available. The unit characterised by X_{Max} is an *IFU* unit if (22) is higher than a critical value, that can be derived from tables originally elaborated by Quesenberry and David (1961). The procedure goes on one unit at a time, excluding units added to *IFUs* at each step from calculations.

3.3 Non-parametric tests based on thresholds

If the empirical score distribution is quite far from normality, we cannot use parametric tests. Among the wide range of available methods, we propose two criteria that can be easily adapted to the problem under discussion.

A first non-parametric test can be based on the *MAD* function (*Mean Absolute Deviation*). When *n* *X*-values are available, MAD_X is the median of the *n* absolute differences $\left|X - q_{(X)0,50}\right|$, where $q_{(X)0,50}$ is the *X* median and represents a less efficient estimator of the population standard deviation, however generally more robust than the sample standard deviation S_X. Sprent (1998) defines a simple and reasonably robust method based on the following rule:

$$(X - q_{(X)0,50})/MAD_X > Max \qquad (23)$$

where *Max* is a critical threshold to be determined. Running test (23) once at a time, functions $q_{(X)0,50}$ and MAD_X must be recalculated at each step, excluding units already detected as *IFUs*. Within an outlier detection frame, Sprent and Smeeton (2001) suggested to assume *Max*=5, since we can consider the empirical relation 5*MAD*=3*S*; if available data - excluded the unit under observation - follow *approximately* a normal distribution, then anomalous values should be more distant than 3*S* from their mean. In an *IFU* context, a lower choice for *Max* could be acceptable, even though this subjectivity is the main limit of the method.

A further non-parametric test is based on the outlier detection procedure proposed by Hidiroglou and Berthelot (1986). A unit will be considered to be an *IFU* if:

$$X > q_{(X)0,50} + \alpha\,(q_{(X)0,75} - q_{(X)0,50}) \qquad (24)$$

where $q_{(X)}$ are the *X*-quantiles already defined in section 2.1 and α is a subjective coefficient. Also in this case, as for (23), the method could be strongly affected by the choice of α, whose level could be quite different from that currently used for outlier detection[6]. Normally, it ranges from 2 to 5.

4. The multivariate case

The identification of *IFUs* can be based on more than one indicator derived from the survey. Indicators can be single variables (as turnover, costs, number of persons employed in the case of business surveys) or specific functions applied to the same variable. If *k* indicators are taken into account, a simple way to proceed simply consists in considering all

[6] As regards the choice of the coefficient α, see also Davila (1992).

those units that turn out to be *IFUs* for *at least one* indicator h as *IFUs*, according to its individual score function Φ_{2hi} and rule (8). This is the *enlarged* criterion, since we should obtain a relatively large number of *IFUs*.

On the other hand, rule (8) could be applied to an average score function defined as:

$$\Phi_{2i} = \sum_{h=1}^{k} \Phi_{2i}^{(h)} P^{(h)} \tag{25}$$

where $P^{(h)}$ is a general coefficient related to the h^{th} indicator. These coefficients are aimed at: a) eliminating the effects due to different magnitudes (and/or different units of measurement) of indicators and ensure the possibility to add them to each other; b) assigning a specific weight to each indicator. In order to obtain a), a simple choice is:

$$P^{(h)} = \overline{\Phi}_2^{(h)} \qquad \text{where:} \qquad \overline{\Phi}_2^{(h)} = n^{-1} \sum_{i=1}^{n} \Phi_{2i}^{(h)} \,. \tag{26}$$

This is particularly useful when we take into account indicators rather than variables as shown in section 5.

If we consider b) as most relevant, the P coefficients can be assumed equal to some weights W, that can be defined according to the relative weight of each indicator on the overall variance[7] or on the basis of more particular rules.

An alternative way to calculate an average score function is based for instance on the following formula, provided that $y_i^{(h)}$ is the value assumed by the i^{th} unit on the h^{th} indicator:

$$\Phi_{2i}^{*} = \sum_{h=1}^{k(i)} \Phi_{2i}^{(h)} W_i^{(h)} \qquad \text{where:} \qquad W_i^{(h)} = \left(y_i^{(h)} \Big/ \sum_{j=1}^{n^{(h)}} y_j^{(h)} \right) \left[\sum_{h=1}^{k(i)} \left(y_i^{(h)} \Big/ \sum_{j=1}^{n^{(h)}} y_j^{(h)} \right) \right]^{-1} \tag{27}$$

and $n^{(h)}$ is the number of units on which the h^{th} indicator can be measured, with $k(i) \le k$. The main difference with respect to (25) is that *different* weights are used for each single unit considered. Also in this case, the sum of weights for each unit i^{th} is equal to one and weights should be estimated using data referred to a previous time (t-1).

The use of (27) could be useful when in the survey unit only $k(i)$ of the k indicators can be measured on each i^{th}, e.g. $y_i^{(h)} \neq 0$ for $h \in H(i)$, where $H(i)$ includes $k(i)$ indicators. A typical example is the monthly survey on industrial production, currently carried out in each developed country. In this case, each observation unit (enterprise, local unit or local "Kind of Activity Unit") can produce one or more industrial products. These $k(i)$ products are generally expressed as different units of measurement and could vary along time. These considerations justify the use of weights as shown in the second equality in (27).

[7] In this case indicators should be previously standardized.

A further criterion can be mentioned: if $H(i)^*$ is the number of indicators for which the i^{th} unit is an *IFU* according to the univariate test based on (8), one can calculate:

$$\left(\sum_{h \in H(i)^*} W^{(h)} \Big/ \sum_{h=1}^{k} W^{(h)} \right) \tag{28}$$

and then verify if (28) – according to a preliminary conversion as in (7) – is higher than a set threshold, on the basis of criteria as described in sections 3.2 and 3.3. In this way we consider the i^{th} unit as an *IFU* by verifying the relative overall magnitude of variables for which this unit is critical according to the univariate score (8). This criterion cannot be used if *different* indicators can not be summed. From a logical point of view, this implies a double application of score functions: the first to the single *y*-values, the second to weights *W* assigned to indicators for each unit.

Another criterion can be obtained by generalising that proposed by Mckenzie (2003, 478). We assume to have observed *k* variables measured along *T* time periods before the period under observation on *n* units. For each variable *h* we can calculate scores $r_i^{(h)}$ (or $\Phi_i^{(h)}$). Then, on the basis of the nx(T-1) available individual scores (thus excluding data referred to the more recent period), we determine deciles of the empirical score distribution. The same procedure is carried out separately for each variable (and, of course, separately in each stratum derived from the original sample design). Then, for each unit we calculate scores referred to the last time *T* and verify, for each variable, which decile group they belong to. Finally, each unit is assigned a priority *IFU* score correspondent to the maximum decile group, where deciles have been assumed to be numbered from 0 to 9 (0 to 0-10$^{\text{th}}$ percentile, 1 to 11$^{\text{th}}$-20$^{\text{th}}$ percentile and so on). For instance, if k=2 and a unit falls in the second decile group for a variable and in the third decile group for another, an *IFU* score equal to 3 will be assigned. Even though relatively simple to be implemented, a certain loss of information must be paid when passing from original data to deciles.

5. A comparison study: the industrial production index[8]

Some of the proposed methods have been applied to a real case, given by the Italian industrial production monthly survey, carried out by ISTAT (IPI: Industrial Production Index).

This is not a true sample survey, being based on a cut-off panel including the most representative enterprises operating in the NACE sections C, D and E. The panel includes about 5,100 enterprises and is quite steady along the whole period from a base year to the new one.

The number of (micro)-products investigated is around 1,100: of course, each unit produces more than one product and provides monthly reports on each item output. The main goal of the survey is the calculation of production indexes – with base 2000=100 – realised each month.

[8] Data used in this section are referred to 2004 and 2005 and, as a matter of fact, do not take into account late evolutions concerning the industrial production monthly survey.

Peculiarities of the survey are both the continuous process of death and birth of new micro-products and the use of different units of measurement for production, depending on the particular product concerned[9]. These constraints led to the calculation of lower level indexes at macro-product level only: actually there are 548 macro-products - that correspond to single micro-products or aggregations of them – and for each of them a single index of change is calculated, according to the sum of productions of those micro-products that refers to each macro-product[10]. Higher order indexes are calculated through weighted arithmetic means: while weights (referred to the base year) are obtained from the production value at macro-product level, they are based on value added for all other higher order indexes.

At the moment, the lag between the end of the reference month and the day of data release is about 40 days as required by the EU Short-term Statistics Regulation (EUROSTAT, 2005*a*). Moreover, rectified data are published one month after the first provisional estimate, taking into account additional late responses. Additional revisions are calculated twice a year.

ISTAT starts receiving monthly data ten days after the end of the reference month and uses data received up to few days (3 to 5) before the press release for the first estimates. Normally data used for the press release cover around 80% of the whole production; revised data are based on an overall coverage exceeding 90%.

In practice, non responses are always *total* non responses. In order to avoid them, a huge effort is made each month for re-contacting late respondents. At this stage of the survey, it is useful to divide the subset of non-responding units into *IFUs* and non-*IFUs*: while data of units in the former group can be estimated according to certain imputation techniques[11], data of units in the latter group can not be estimated and should be therefore obtained according to an intensive *ad hoc* reminder action.

5.1 The concept of IFU in the IPI survey

The main question is: what is an *IFU* in industrial production surveys? Normally the degree of importance of a statistical unit is linked to how much of the overall amount of the target variable is due to that unit: however, this single criterion could not be enough or could be partially misleading for short term statistics.

First of all, we must consider that *IFUs* not always exist: that is the case when, inside a given domain, production is quite homogeneous among units and, as a consequence, the degree of concentration is low: an extreme case is to be found when n units have each $1/n$ of the global production, so that searching for particular *IFUs* could appear to be a nonsense.

On the other hand, a reasonable property for *IFUs* is that, in the extreme case when only their data can be used to carry out estimates, it should be possible to obtain estimates for each estimation domain. As a consequence, at least one unit should respond for each estimation domain.

[9] The questionnaire is quite simple, being based only on one variable (volume of monthly production), expressed in units of measurement depending on the type of product considered. For a part of products (weighting around 11%) the activity is measured through the value of production deflated by a proper production price index; for another part (weighting around 6%) production is measured by the number of hours worked.

[10] For instance, the macro-product “Linen for the house” is composed by the 4 following micro-products: Linen for beds, Linen for tables, Linen for kitchen and toilette, Linen for curtains.

[11] For more details see Gismondi *et al.* (2005).

Moreover, an additional operational constraint is that the number of *IFUs* should be compatible with the real possibility to organise follow ups for them in a short time, so that it cannot be too large.

Finally, the algorithm used for identifying *IFUs* should be as much as possible objective and it should be replied over time in order to verify steadiness or changeability of *IFUs*. In particular, a further technical aspect occurring in short-term surveys is that, in theory, for each infra-annual survey occasion (month, quarter) a different subset of *IFUs* could exist, while, on the other hand, a unique subset could be reasonably managed each month by personnel in charge of reminders for late or non-respondents.

An additional remark concerns what has been already introduced in section 2: when a change is the main object of estimation (as in the case of the IPI survey), both magnitude and variability of individual data along time should be considered, as it has been commented on in section 3.

In a previous work (Gismondi, 2006), we proposed a particular strategy for detecting influent units in the frame of the IPI survey. It was based on a two-stage approach. An analysis was carried out at micro-product level at the first stage and at macro-product level at the second stage. Remembering that a macro product j is determined by one or more micro-products r, the final proposal was based on the idea of identifying those units that turned out to be influential at micro or macro-level as definitive *IFUs*.

A detailed presentation about possible techniques for detecting *IFUs* at macro-level is presented in section 5.2. On the other hand, we will not deal extensively with criteria for detecting *IFUs* at micro-level (for which we address again to Gismondi, 2006). However, it can be helpful to resume shortly the main logic underlying this first step. If ${}^{r}W_{bi}$ is the weight – in base year b – of the unit i on the production of micro-product r, ${}^{r}W_{b[i]}$ is the analogous weight but referred to the unit that, in the decreasing rank of weights, is located at the i^{th} place, and R is the macro-product to which micro-product r belongs, then the set of 3 rules for detecting *IFUs* at micro-level is given under the following conditions:

1) $\sum_{i=1}^{n} {}^{r}W_{b[i]} \geq 0{,}5$ for each micro-product r (29a)

2) ${}^{r}W_{b[i]} \geq 0{,}20$ for at least one unit (29b)

3) Each macro-product R must be characterised by at least one *IFU*. (29c)

According to (29a), for a given micro-product r the n *IFUs* are those units ensuring a coverage of at least 50% of production. However, according to (29b) we should have at least one unit with at least 20% of production (otherwise the search for influential units could not have a real sense) for each micro-product. If this condition is not satisfied, no *IFU* is assigned to micro-product r, even though macro-product R including micro-product r should be characterised by at least one *IFU* (otherwise no index for this macro-product can be calculated, condition 29c).

5.2 Risk function calculation and compared criteria

In the IPI survey, there are two main peculiarities that need to be better focused with respect to the general introduction of the risk function seen in section 2:

1) in the survey both statistical units and products are analysed, so that for each unit a separate risk could be evaluated for each (macro)-product concerned with the unit's production;
2) it is a short-term survey, so that in theory there should be a separate risk function for each month (and, as a consequence, different *IFUs* should be found for each month as well).

The main consequence of the first point is that each elementary risk function should be labelled according to both the particular unit considered (i) and all the macro-products that can be referred to it (R).

According to point 2), it is clear that operationally different subsets of *IFUs* for each distinct month (m) cannot be easily managed, so that it is reasonable to look for a unique set of *IFUs* valid at least for one whole year (Y).

On the basis of the previous requirements, a possible score function for the i^{th} unit, product R and month m in year Y is given by:

$$\Phi_{1,YmiR} = \frac{\left| {}^{R}I_{Ym/b} - {}^{R}I_{Ym/b(-i)} \right|}{{}^{R}I_{Ym/b}} \tag{30}$$

where I is an index number with base 2000=100 and ($-i$) means exclusion of the i^{th} unit from calculations. It is equivalent to function (6) where each sample weight is equal to one, U=0 and option e) given by (11) is used, where we assume $\gamma_i = {}^{R}I_{Ym/b}$ for each i, obtaining this index by using all the available units: even though this additional step is not necessary in order to ensure a statistical meaning to the risk function, it is convenient to include it in (30) in order to better synthesize monthly risks. In particular, the risk function for the i^{th} unit is a weighted average of the risk functions by NACE class, given by:

$$\Phi_{1,Ymi} = \frac{\sum_{R} \Phi_{1,YmiR} W_R}{\sum_{R} W_R} \tag{31}$$

where W_R is the weight of class R in the base year and is given by the value of production or by value added.

Finally, the score function for the i^{th} unit is given by the simple arithmetic mean of all the monthly score functions considered:

$$\Phi_{1,i} = \frac{\sum_{Y=1}^{Y_0} \sum_{m=1}^{12} \Phi_{1,Ymi}}{12\, Y_0} \tag{32}$$

where the first sum is considered over the years (starting from the year "1", they could be more than one, say Y_0). The need to calculate monthly risk average justifies the use of a function as (30), where the division by ${}^{R}I_{Ym/b}$ avoids the risk to weight single monthly risks in different ways because of seasonality of production.

Even though function (30) is itself based on the idea to evaluate changes (C) instead of levels (an index number is itself an indicator of change with respect to a base year), another approach has been tested, using the alternative score function given by:

$$\Phi *_{1,YmiR} = \frac{\left| {}^{R}C_{Ym/b} - {}^{R}C_{Ym/b(-i)} \right|}{{}^{R}C_{Ym/b}} \tag{33}$$

where ${}^{R}C_{Ym/b} = \frac{{}^{R}I_{Ym/b} - {}^{R}I_{(Y-1)m/b)}}{{}^{R}I_{(Y-1)m/b}}$. The compared criteria for identifying *IFUs* are reported in the following summarising scheme:

Formula	Definition	Conversion (7)
(19)	Empirical (pseudo) bias – 0.002%, 0.005%, 0.01%, 0.2%, 0.5% thresholds	No
(19)	Empirical (pseudo) bias – 1%, 3% and 5% thresholds	Yes
(21)	Standardized normal distribution – 2.32, 1.96 and 1 percentiles	Yes
(22)	Grubbs test (extreme studentized deviate) – 3, 3.5 and 4 options	Yes
(23)	Sprent test (Sprent and Smeeton, MAD test) – 4, 5 and 6 thresholds	Yes
(24)	Hidirogluou-Berthelot – α=2, α=3 and α=5 options	Yes

All criteria have been implemented on the basis of function Φ_2 defined by formula (7) except the first one, that was applied without this conversion just to highlight its usefulness. One can note that methods based on bias ratio and empirical bias could not identify any *IFU*, because of the intrinsic meaning of functions (18) and (19) on which they are based.

5.3 Main results

Table 1 shows the number of IFU units (and the corresponding percentage) detected by using two different score functions and six combinations of identification criteria with reference to the years 2004 and 2005. A first result is that, in 2004, the number of *IFU* units is equal to 1,109 using function (30) (level) and to 1,120 using function (33) (change) by a 1% threshold; the same figures for 2005 are equal to 1,092 and 1,118 respectively.

Table 1 - Main results obtained using function Φ_1 with various criteria in the monthly survey on industrial production – 2004 and 2005

CRITERION	Number of *IFU* units				Percentage on the whole sample			
	Function (30)		Function (33)		Function (30)		Function (33)	
	2004	2005	2004	2005	2004	2005	2004	2005
(19) – 0.002%	1,835	1,795	4,626	4,829	35.2	34.6	88.6	93.2
(19) – 0.005%	920	890	4,334	4,513	17.6	17.2	83.0	87.1
(19) – 0.01%	471	444	3,954	4,080	9.0	8.6	75.7	78.7
(19) – 0.2%	5	3	1,026	1,050	0.1	0.1	19.7	20.3
(19) – 0.5%	1	0	444	446	0.0	0.0	8.5	8.6
(19) – 1% (*)	1,109	1,092	1,120	1,118	21.2	21.1	21.5	21.6
(19) – 3% (*)	456	451	496	491	8.7	8.7	9.5	9.5
(19) – 5% (*)	280	260	296	293	5.4	5.0	5.7	5.7
(21) – 2.32	62	93	41	40	1.2	1.8	0.8	0.8
(21) – 1.96	76	109	56	50	1.5	2.1	1.1	1.0
(21) – 1	173	244	136	133	3.3	4.7	2.6	2.6
(22) – 3	1,003	1,010	1,118	1,104	19.2	19.5	21.4	21.3
(22) – 3,5	657	574	696	683	12.6	11.1	13.3	13.2
(22) – 4	440	420	486	451	8.4	8.1	9.3	8.7
(23) – 4	981	966	1,039	1,012	18.8	18.6	19.9	19.5
(23) – 5	835	830	890	856	16.0	16.0	17.0	16.5
(23) – 6	721	706	774	748	13.8	13.6	14.8	14.4
(24) – 2	823	826	831	846	15.8	15.9	15.9	16.3
(24) – 3	597	574	622	623	11.4	11.1	11.9	12.0
(24) – 5	372	349	396	380	7.1	6.7	7.6	7.3

(*) After conversion (7). *Source*: processing of ISTAT data.

Generally speaking, the largest differences among the final number of *IFUs* do not derive from the use of different score functions, but rather from the particular identification criterion chosen. The only exception occurs when a non-standardised function is used, as in the first 5 rows of the table based on the empirical (pseudo) bias without conversion (7) that symmetries its empirical density curve.

A fundamental result is that the use of a risk function as (33), based on change rather than level evaluation, implies a larger number of critical units. That does not necessarily imply that a larger percentage of *IFU* units that are common to the years 2004 or 2005 is guaranteed by function (33) instead of function (30), as it may be noted according to the next table 2.

The stability of the *IFUs* subset over time is a relevant feature of the follow-up process used in short-term surveys. However, it can be helpful to evaluate how the number of *IFU* units changes with respect to different thresholds. Moreover, it should be verified if the final number of *IFUs* is sustainable according to operational constraints concerning the number of persons engaged in the follow-up process, time deadlines for publication of provisional data, etc..

According to these needs, one can divide the compared criteria into 6 groups:

1) the pseudo bias criterion (19) *without* conversion (7) should be avoided, because – as already remarked – it leads to a quite unsteady number of *IFU* units depending on the threshold or the reference year used.
2) The pseudo bias criterion (19) with conversion (7) could be considered one of the most suitable for the purpose under study: the number of IFU units detected is quite similar comparing 2004 with 2005 – especially when risk function (33) is used – and relatively small differences occur when we compare risk functions (30) and (33) as well. Moreover, the use of a 1% threshold leads to a number of IFU units quite similar to that actually used in the IPI survey frame[12].
3) The standardized normal curve criterion (21) is probably not very useful as well: it identifies a quite low number of *IFUs* in comparison with the others; moreover, this amount changes too much depending on the threshold used, on the year of reference – when function (30) is used – and on the particular risk function adopted. This unsatisfactory performance is probably due to the fact that the empirical density curve of standardised risks is far from normality and is still characterised by a positive asymmetry. As a consequence, using standardised normal percentiles we find a quite low number of critical units.
4) The Grubb test (22) could be seen as the second best approach with respect to the pseudo bias commented above: as a matter of fact, the number of *IFUs* detected with a threshold equal to 3 is not very far from that obtained with the pseudo bias and a 1% threshold. The number of *IFUs* changes more over time than by using the pseudo bias; on the other hand, it is less responsive to the use of different thresholds, even though the thresholds selected in the application are subjective.
5) The Sprent test (23) allows to detect a number of *IFUs* similar to those used in the current survey when a 4% threshold is used and is itself quite steady although the year or the risk function considered may change. It must be remarked that a larger number of *IFUs* is found when risk function (33) is adopted.
6) Finally, the Hidiroglou-Berthelot criterion (24) produces a lower number of *IFUs* with respect to the other standardised criteria seen above, but is itself quite reliable and allows to obtain more steady results when risk function as (33) is adopted.

The right part of the table shows the percentage of *IFU* units on the whole sample, even though we should remember that additional *IFUs* detected at micro-product level have not been taken into account, according to what already said in section 5.1.

Generally speaking, these results are similar to those obtained in a previous work (Gismondi, 2007), where the empirical application concerned retail trade data. This may depend on similarities between the empirical density curves of monthly industrial production and retail trade turnover.

[12] As already mentioned in section 5.1, the *IFU* units effectively used in the IPI survey framework are identified on the basis both of macro-level and micro-level approaches (for more details, see for instance Gismondi *et al*., 2005). In short, the real *IFUs* adopted in the IPI survey context are, normally, large units (in terms of amount of production in the base year) whose longitudinal profile is analysed along several months (compare, for instance, Pietsch, 1995).

Additional features of the various criteria compared have been reported in table 2. The first 2 columns represent the percentage of critical units found by both risk functions (30) and (33) and over time (comparing 2004 and 2005).

If we consider 2005 data, we see that for all criteria with conversion (7) - except the standardised normal (21) – the share of units detected as *IFUs* using (30) and (33) ranges from about 40% to 50% depending on the threshold and the criterion used. As a consequence, the choice of the risk function plays a relevant role not only to establish the *overall number* of critical units, but to identify *which* of them are critical.

Moreover, the last 2 columns show that the share of units that – given the risk function used – is critical in both years is quite larger when using (30) rather than (33), because the largest shares obtained are about 70% in the former case and about 50% in the latter. The use of (30) can thus ensure more steady results over time.

This result probably depends on the larger variability of values assumed by function (33) – that by definition is based on changes between two following years – with respect to those of function (30), based on (more steady) amounts of the y-indexes. Since the subset of *IFU* units should remain quite steady along time – in order to ensure the possibility to build up an efficient intensive follow up system – probably the better operational solution should be in favour of function (30).

Table 2 - Some indicators comparing IFU units obtained using function (30), function (33) and IFU units actually used in the monthly survey on industrial production–2004 and 2005

CRITERION	% intersection between *IFU* units obtained with functions (30) and (33)		% intersection between *IFU* units in 2004 and 2005	
	2004	2005	Function (30)	Function (33)
(19) – 0.002%	39.7	37.2	77.8	92.1
(19) – 0.005%	21.2	19.7	69.5	88.7
(19) – 0.01%	11.9	10.9	66.2	84.2
(19) – 0.2%	0.5	0.3	14.3	52.0
(19) – 0.5%	0.2	0.0	0.0	41.1
(19) – 1% (*)	53.2	53.2	70.1	53.8
(19) – 3% (*)	42.1	42.3	66.3	42.5
(19) – 5% (*)	38.1	36.5	66.0	38.0
(21) – 2.33	14.4	13.7	46.2	23.1
(21) – 1.96	15.8	18.7	45.7	25.0
(21) – 1	26.1	24.8	53.7	34.0
(22) – 3	52.7	51.4	69.1	53.8
(22) – 3.5	44.7	45.3	66.0	46.6
(22) – 4	41.4	41.6	65.6	40.5
(23) – 4	51.8	50.5	68.8	51.8
(23) – 5	49.7	48.9	68.8	48.7
(23) – 6	47.3	46.9	67.5	47.4
(24) – 2	49.5	49.0	68.6	48.4
(24) – 3	42.6	45.3	65.9	45.3
(24) – 5	39.9	39.7	65.2	36.9

(*) After conversion (7). *Source*: processing of ISTAT data.

6. Conclusions

When treating non–responses, non–response prevention is sometimes under-estimated or treated according to not fully appropriate criteria (Kalton *et al*., 1989).

Generally speaking, the identification of critical units that should be object of a priority follow-up system in case of non response depends on: 1) the particular individual score function adopted, estimating the risk due to non- availability of a unit for estimates; 2) the criterion used to detect critical values of the score function and identify *IFUs*.

Both aspects have been re-analysed according to the most currently used procedures and some new proposals, by using a comparative exercise referred to a real short-term estimation process for which timeliness of decisions is fundamental.

The underlying idea is that statistical relevance and degree of coverage are related concepts, that however should be kept *separated*. The *IFU* feature is an *intrinsic* character of statistical units; techniques to identify critical units should be based not only on a procedure driven by coverage only: critical units are automatically detected through a simple decreasing ranking and the selection of all the first units in the rank ensuring a certain coverage level.

On the other hand, using a criterion allowing to detect a lower number of *IFUs* – thus consequently providing a relatively low *IFU* coverage – should be preferred if it is based on a rational strictly connected with the evaluation of estimate error (Granquist and Kovar, 1997).

Empirical results in the light of the techniques compared show that the most delicate aspect to be carefully evaluated is the choice of the criterion to detect critical units rather than the possibility to build up the individual score functions in different ways. Different criteria lead to very different numbers of *IFUs*, and we could verify that the decrease of estimate error is less than proportional with respect to the increase of *IFUs*.

In this context, when the estimation of a change is the main purpose of a survey, a critical aspect is the choice between a risk function based on level or change: while the first option seems to ensure more steady results over time, the second determines a larger number of *IFUs*, e.g . implying a more precautionary follow-up strategy.

By steady conditions, the final choice should be probably in favour of criteria strictly linked to features of estimator and sample design, as bias ratio and empirical pseudo bias defined in section 3.1.

The final decision on the most suitable criterion to detect *IFU* units is a double-faced problem: we should use a proper statistical method, while taking into account organisational constraints, as the largest number of units that could be object of a reminder within a reasonable time.

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Multiplier Decomposition, Inequality and Poverty in a SAM Framework[1]

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Abstract

The aim of this paper is to show how and why it is possible to assess both direct and indirect effects of exogenous income injections on mean income of different household groups using a new approach based on the decomposition of SAM-based multipliers. The approach we propose allows analyzing the level of inequality in the distribution of income linking the formation of household income to the features of country's productive structure. After deriving the "accounting price multipliers matrix", we introduce a new technique (Pyatt, Round, 2006) in order to decompose each element of the total multiplier matrix which enlighten in "microscopic detail" the linkages between each household group's income and other accounts whose income has been exogenously injected. The meaning and the relevance of the multiplier decomposition method will be illustrated with an application to the Italian economic system.

Keywords: Income distribution, social accounting matrix, multiplier decomposition, growth, labour market, structure of production.
JEL: D31, D33, D57, O15, O43.

1. Introduction

The relationships among growth, inequality and poverty have been widely explored in the last years with different approaches. International organizations, national governments and civil society have been increasingly committed to fight against poverty. In order to set up poverty and redistributive policies, the definition of poverty profiles and the measurement of the impacts on poverty of economic growth, at an aggregated and sectoral level, should be assessed. Traditionally poverty and inequality are considered essentially as a microeconomic issue. Poverty profiles or inequality determinants are related to individual features. However, the impact of economic policies is related to the macroeconomic and structural policies, i.e. aggregate economic variables. Therefore both microeconomic and macroeconomic approach should be adopted.

In order to improve the political targeting at the macro level and to better evaluate the role of growth in the poverty reduction strategies the so-called growth-poverty-inequality

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triangle has been proposed by Bourguignon (2003, 2004). This strand of literature not only investigate the role played by growth and inequality in reducing poverty, but tries to identify a causal relationship between micro and macro variables. This analysis is undertaken mostly at an empirical level. A better understanding of these nexus, however, requires analysing the links among income distribution by factors shares, personal income distribution and alternative policies.

A better understanding of the relationships between income distribution in different Households groups and alternative policies requires to build a system in which the information's on production, intermediate and final demand and income distribution between and inside different groups are linked together. The Social Accounting Matrix (SAM) is the schema for this goal. The inclusion in the SAM of data related to the production side and of data related to the income distribution and to consumption expenditure allows to consider the SAM not only as a database and as an accounting tool, but also, in a wider sense, as a macroeconomic simulation model.

This strand of literature can be considered complementary to the traditional one, since it allows relating the formation of individual/family income to the characteristics of the productive structure of each country. "The impact of a sector's output on poverty alleviation can be direct through the increase in incomes accruing to the poor households who contributed through their labour or land to the sector's growth of output. But another part of poverty alleviation results from the indirect effects operating through the interdependence of economic activities, i.e. the closed loop effects familiar in the Social Accounting Matrix (SAM) literature" (Thorbecke, Jung, 1996, p. 280). This kind of effect has been often ignored by current literature on poverty, income distribution and growth.

The approach we propose can be used for structural analysis of the features of the economic system and for the analysis of the effects of pro-growth and antipoverty policies. The SAM can be used as a Leontief linear model, once we introduce the hypothesis of constancy for the coefficient of income distribution and of expenditure. The solution of the model brings to a matrix of multipliers, which allows assessing the effects of changes of some of the variables (exogenous) on the others (endogenous) of the system. In order to estimate the changes in the incomes of different groups (deciles or socio-economic groups) it is possible to adopt the multiplier decomposition approach based on a Social Accounting Matrix (SAM).

Starting from the seminal Pyatt and Round's decomposition method of "accounting multipliers matrix" (Pyatt, Round, 1979) we will determine the global multipliers values of different households groups. This decomposition allows measuring the change in the level of income of each endogenous group affected by a change in the incomes of the exogenous accounts that are included in the SAM. This can be considered a first step in order to link changes in the level of poverty and policy measures.

The second step will be to decompose each total multiplier' element in order to enlighten in "microscopic detail" the linkages between the incomes of each socio-economic group with the other accounts. In particular, it is interesting to assess the linkages with the activities and with the factors, i.e. the linkages between the household endowment and the features of the productive system.

2. The SAM as a simulation model and the decomposition of the multiplier matrix

A SAM has frequently been used to examine the partial equilibrium consequences of real shocks, using a multiplier model that treats the circular flow of income endogenously. "If a certain number of conditions are met - in particular, the existence of excess capacity and unemployed or underemployed labour resources - the SAM framework can be used to estimate the effects of exogenous changes and injections, such as an increase in the demand for a given production activity, government expenditures or exports on the whole system. As long as excess capacity and a labour slack prevail, any exogenous change in demand can be satisfied through a corresponding increase in output without having any effect on prices. Thus, for any given injection anywhere in the SAM, influence is transmitted through the interdependent SAM system. The total, direct and indirect, effects of the injection on the endogenous accounts, i.e. the total outputs of the different production activities and the incomes of the various factors and socioeconomic groups are estimated through the multiplier process" (Thorbecke, 2000, p. 17).

In order to measure the effects occurring in some variables (the exogenous ones) on the other (the endogenous ones) of the system a very aggregated SAM (Figure 1) must be introduced. A main outcome of SAM-based multiplier analysis is to examine the effects of real shocks on the economy on the distribution of income across different groups of households. "One other important feature of SAM-based multiplier analysis is that it lends itself easily to decomposition, thereby adding an extra degree of transparency in understanding the nature of linkage in an economy and the effects of exogenous shocks on distribution and poverty" (Round, 2003, p. 271).

The determination of a multi-sector income multiplier is a distinguishing characteristic of the models based on the SAM. The equilibrium solution is obtained following the same procedure as in the input-output analysis and using the SAM as a linear model. "It is obvious that the SAM formulation contains more information and a higher degree of endogeneity since it captures the endogenously derived effects of income distribution on consumption, which the Leontief national model does not" (Thorbecke, 2000, p. 22).

The multiplier approach allows quantifying the different ways by which an income equally earned by each socio-economic group identified in the Household sector, turns into different disposable income levels through the three stages of spending, production and redistribution. The accounting multipliers obtained using the SAM as a linear model allow capturing the structural features of the income distribution and the interrelations between different households groups. The resulting inequality in personal income distribution can be considered as the minimum inequality compatible with the given productive and spending structures, and hence as a result of the mechanism only explicitly considered in the model.

The income distribution of the Institution Households in the SAM must be considered as an equilibrium one, i.e. the distribution that assure the balance between the final demand for consumption and the supply of different commodities from the productive sectors in a given year. In this SAM the endogenous components (Activities, Factors and Private Institutions as Households and Companies) can be isolated from the exogenous ones (Government, Rest of the World and Capital/Saving) by aggregating one or more matrices of the SAM. "A truncated SAM consolidates all exogenous transactions and corresponding leakages and focuses exclusively on the endogenous transactions and transformations" (Thorbecke, 2000, p. 18). Our model, in particular, assumes that the consumption demand comes only by the Household sector. Private Companies receive income from Factors and redistribute it to other Private Institutions.

This equilibrium distribution, and the corresponding level of inequality, can be determined as a solution of the SAM once it is considered as a linear model. Following a Keynesian approach, we can assume that the total level of income of each socio-economic group determines the level of consumption of different commodities by the Institution Households. The equilibrium solution through the SAM determines the income distribution of the Private Institutions consistent with a given production structure under the assumption that the final demand depends on the disposable income of the Endogenous Institutions.

The traditional input-output analysis based on multipliers assumes the consumption demand as exogenous and the output of various activities depending on the propensities of final demand so that the composition of demand influence that of the value added. The opposite is not true because the input-output model does not include the link between the value added and the primary income distribution to the different Households groups. In the SAM model, on the opposite side, Households groups' incomes assume different values depending on the composition of final demand. This happens because our model takes into account the structure of personal income distribution as depending on the composition of the value added as determined by the structure of production.

Figure 1 - Exogenous and endogenous accounts in a simplified SAM.

	Endogenous Accounts				
	Activities	Factors	Private Institutions	Exogenous Accounts	Total
Activities	$\mathbf{S_{11}}$	**0**	$\mathbf{S_{13}}$	$\mathbf{x_1}$	$\mathbf{t_1}$
Factors	$\mathbf{S_{21}}$	**0**	**0**	$\mathbf{x_2}$	$\mathbf{t_2}$
Private Institutions	**0**	$\mathbf{S_{32}}$	$\mathbf{S_{33}}$	$\mathbf{x_3}$	$\mathbf{t_3}$
Exogenous Accounts	$\mathbf{l'_1}$	$\mathbf{l'_2}$	$\mathbf{l'_3}$	$\mathbf{x_4}$	$\mathbf{t_4}$
Total	$\mathbf{t'_1}$	$\mathbf{t'_2}$	$\mathbf{t'_3}$	$\mathbf{t'_4}$	

The matrices of expenditure $\mathbf{A_{jk}}$ are obtained dividing each element in the transaction matrices of endogenous accounts $\mathbf{S_{jk}}$ by the correspondent column sum vectors $\mathbf{t'_k}$. The hypothesis of fixed expenditure coefficients resulting from $\mathbf{A_{jk}}$ is consistent with the assumptions of the linear expenditure system developed by Stone for which there is widespread empirical support (Stone, 1954). The matrix $\mathbf{A_{jk}}$ is obtained dividing the matrix $\mathbf{S_{jk}}$ by the diagonal matrix $\hat{\mathbf{t}}_{\mathbf{k}}$ whose elements are the components of $\mathbf{t'_k}$.

$$\mathbf{A_{jk}} = \mathbf{S_{jk}}\left(\hat{\mathbf{t}}_{\mathbf{k}}\right)^{-1} \qquad (1)$$

The normalisation of the transaction matrices $\mathbf{S_{jk}}$ allows the constraints relating to row and column totals of the SAM in Figure 1 to be rewritten isolating the group of the r (three in our case) endogenous accounts from the exogenous ones. We can, thus, write

$$\mathbf{t} = \mathbf{A\ t} + \mathbf{x} \qquad (2)$$

$$\mathbf{t_4} = \mathbf{l'_1 t_1} + \mathbf{l'_2 t_2} + \mathbf{l'_3 t_3} + \mathbf{x_4} \qquad (3)$$

Equation (3) indicates that the equilibrium position of the accounts relating to exogenous Institutions is achieved once endogenous accounts are in equilibrium. The formulation in equation (2) indicates that vector **t** of receipt totals for each endogenous account can be obtained from vector **x**, expressing the receipt totals of exogenous Institutions, by the generalised inverse **A**.

With reference to the SAM of Figure 1 equation (2) can be written out in explicit form as:

$$\mathbf{t}_1 = \mathbf{A}_{11}\,\mathbf{t}_1 \qquad + \mathbf{A}_{13}\,\mathbf{t}_3 + \mathbf{x}_1 \tag{4}$$

$$\mathbf{t}_2 = \mathbf{A}_{21}\,\mathbf{t}_1 + \qquad\qquad + \mathbf{x}_2 \tag{5}$$

$$\mathbf{t}_3 = \qquad \mathbf{A}_{32}\,\mathbf{t}_2 + \mathbf{A}_{33}\,\mathbf{t}_3 + \mathbf{x}_3 \tag{6}$$

Which yields:

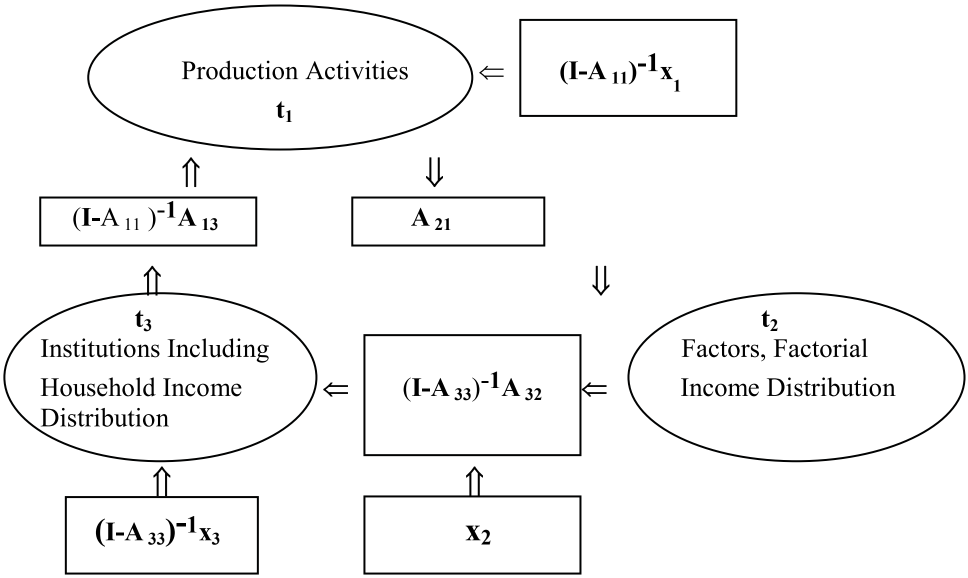
$$\mathbf{t}_1 = (\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{x}_1 + (\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{A}_{13}\,\mathbf{t}_3 \tag{7}$$

$$\mathbf{t}_2 = \mathbf{A}_{21}\,\mathbf{t}_1 + \mathbf{x}_2 \tag{8}$$

$$\mathbf{t}_3 = (\mathbf{I}-\mathbf{A}_{33})^{-1}\mathbf{x}_3 + (\mathbf{I}-\mathbf{A}_{33})^{-1}\mathbf{A}_{32}\,\mathbf{t}_2 \tag{9}$$

This last set of relationships, following Thorbecke (2000), can be represented graphically as in Figure 2.

Figure 2 - Multiplier Process among endogenous accounts



The loop representation of Figure 2 shows clearly and explicitly the mechanisms through which the multiplier process operates as the result of different exogenous injections, taking in account that:

$\mathbf{x}_1$ = exogenous commodities' fnal demand from government consumption, export and investment demand

$\mathbf{x}_2$ = exogenous factors' final demand for factors from government consumption, export and investment demand

$\mathbf{x}_3$ = exogenous injection from government transfers, and remittances from abroad toward the Private Institutions.

Thus let's start with an exogenous increase (injection) of export, government consumption, or investment demand $\mathbf{x}_1$. This generates a rise in the output of the corresponding production activity of $(\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{x}_1$. In turn, the additional factors of production which have to be employed to create the additional output generate a stream of value added $\mathbf{A}_{21}\mathbf{t}_1$ which becomes income from factors in addition to any exogenous factor income received from other regions or from abroad and from the government, namely $\mathbf{x}_2$.

In the next link, Households (and Companies) receive income based on their resource endowment ($\mathbf{A}_{32}$) and transfers system ($\mathbf{A}_{33}$) as well as exogenous government subsidies and transfer payments and remittances from other regions and abroad, i.e. $(\mathbf{I}-\mathbf{A}_{33})^{-1}\mathbf{x}_3$. Finally, the triangle is closed through the pattern of household (and companies) expenditures on commodities which translates into new production and in a corresponding additional flow of income accruing to production activities equal to $\mathbf{t}_1 = (\mathbf{I}-\mathbf{A}_{11})^{-1}\ \mathbf{A}_{13}$.

The formulation in Figure 2 can be considered a generalization of the Leontief model because it takes in account also the effects of personal income distribution vector $\mathbf{t}_3$ on the consumption of the various socio-economic groups through the matrix $\mathbf{A}_{13}$ which expresses the consumption pattern of each socio-economic group of Households. In the traditional Leontief model Households' consumption is included in the final demand vector as an exogenous component and the multiplier process can be expressed as follows: $\mathbf{t}_1=(\mathbf{I}-\mathbf{A}_{11})^{-1}\mathbf{x}_1$ where $\mathbf{A}_{11}$ is the input-output coefficient matrix and $\mathbf{x}_1$ is exogenous total final demand.

The circular flow and the multiplier effects can be derived also starting from the equilibrium conditions expressed in equations (2) and (3). These conditions allow that only equation (2) is taken into consideration and it is rewritten as

$$\mathbf{t} = (\mathbf{I} - \mathbf{A})^{-1}\ \mathbf{x} = \mathbf{M}\mathbf{x} \qquad (10)$$

$$\mathbf{M} = (\mathbf{I} - \mathbf{A})^{-1} \qquad (11)$$

Thus, from (10), endogenous incomes $\mathbf{t}$ (i.e. production activity incomes, $\mathbf{t}_1$, factors incomes, $\mathbf{t}_2$, and Institutions' incomes, $\mathbf{t}_3$ as shown in Figure 1) can be derived by premultiplying injection $\mathbf{x}$ by a multiplier matrix $\mathbf{M}$. This formulation indicates that the vector $\mathbf{t}$ of receipt totals for each endogenous account can be obtained from vector $\mathbf{x}$, expressing the receipt totals of exogenous Institutions, by the generalised inverse $\mathbf{A}$.

This matrix **M,** introduced by Pyatt and Round (1979) in a seminal contribution, has been referred to as the accounting multiplier matrix because it explains the results obtained in a

SAM and not the process by which they are generated. This accounting multipliers matrix can be interpreted as a simplified model of the actual way the system is working. From another side, the results of the multiplier analysis can be interpreted as a demonstration of how the economic system is expected to behave in case the model assumptions perfectly reflect the real situation: any possible deviation from reality would then indicate both the correct parts and those which must be better calibrated. (Round, 2003).

"Accounting multipliers" are derived in constant prices and they are therefore "fixed-price" in a formal sense. They show average responses of endogenous variables to exogenous injections. One limitation of the accounting multiplier matrix is that "it implies unitary expenditure elasticities" (Thorbecke, 2000, p. 19). The prevailing average expenditure propensities in **A** are assumed to apply to any incremental injection. Average responses could be different from marginal ones. Then a matrix of 'fixed-price multipliers', based on marginal responses, could be introduced. "The distinction simply recognises that the marginal responses in the system, even in a fixed-price world, may be different from what they are on average" (Round, 2003, p. 14). The estimate of the value of expenditures elasticity should be obtained only comparing the SAM values obtained for different years or with econometric methods.

M in equation (10) is the matrix of the global multipliers and shows the overall effects resulting from the direct and indirect transfer processes generated by an initial increase in anyone of the three exogenous components $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ on each element of the *r* (in our case three) endogenous accounts. Following Pyatt and Round (1979) and Bottiroli Civardi (1988, pp. 94-102) it is possible to decompose the multiplier matrix **M** into three components $\mathbf{M}_1$, $\mathbf{M}_2$ and $\mathbf{M}_3$. This decomposition has an economic meaning for a structural analysis of income distribution, inequality and poverty, among and inside the Private Institutions, with particular reference to the Households' groups.

Equation (2) can be reformulated as

$$\mathbf{t = A\,t + x = A\,t + A_0\,t - A_0\,t + x = M_1\,(A\text{-}A_0)\,t + M_1\,x} \qquad (12)$$

Where matrix $\mathbf{A_0}$ is defined as:

$$\mathbf{A_0} = \begin{vmatrix} \mathbf{A}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{33} \end{vmatrix} \quad \text{and where:}$$

$$\mathbf{M_1} = (\mathbf{I\text{-}A_0})^{-1} = \begin{vmatrix} (\mathbf{I}-\mathbf{A}_{11})^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (\mathbf{I}-\mathbf{A}_{33})^{-1} \end{vmatrix} = \begin{vmatrix} {}_1\mathbf{M}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & {}_1\mathbf{M}_{33} \end{vmatrix} \qquad (13)$$

The $\mathbf{M}_1$ multiplier matrix captures the transfer elements. It expresses the effects within each endogenous account generated by direct transfers that are independent from the closed-loop process of income through the system. If we consider an exogenous injection of income in one endogenous account of the three blocks of the matrix, multiplier matrix $\mathbf{M}_1$ evaluates the impact on accounts belonging to the same block (for example, activities) due only to transfer effects within the same block. We can then refer to $\mathbf{M_1}$ as within group or transfer multiplier. The multiplier matrix $\mathbf{M_1}$ is a diagonal block matrix where the first diagonal block expresses the multiplier effects of the transfers within the activities and it is precisely the Leontief's inverse matrix. Since it is assumed that no direct transfers between factors take place, second diagonal block in $\mathbf{M_1}$ is the identity matrix $\mathbf{I}$. The third block captures the multiplier effects due to the transfers between endogenous Institutions.

The definition of $\mathbf{M_1}$ allows to introduce matrix $\mathbf{A^*}$ as $\mathbf{M_1 (A-A_0) = (I-A_0)^{-1}(A-A_0)}$ where:

$\mathbf{A^*_{13} = (I- A_{11})^{-1} A_{13}}$

$\mathbf{A^*_{21}= A_{21}}$

$\mathbf{A^*_{32}= (I- A_{33})^{-1}A_{32}}$ or, if $\mathbf{A_{33}= 0}$, $\mathbf{A^*_{32}= A_{32}}$

The elements of $\mathbf{A^*}$ generate the circular flow of income. If we assume that $\mathbf{(I-A^*)^{-1}}$ exists, we can rewrite equation (12) as:

$$\mathbf{t = [(I-A^*)^{-1} M_1] x = (I-A^*)^{-1} (I-A_0)^{-1} x = Mx} \tag{14}$$

Equation (14) provides an initial decomposition of the matrix $\mathbf{M}$ into a transfer effects matrix $\mathbf{(I-A_0)^{-1}}$ and a complementary matrix $\mathbf{(I-A^*)^{-1}}$ that can be further decomposed if we express it as:

$$\mathbf{(I- A^*)^{-1} = (I- A^{*r})^{-1} (I + A^* + A^{*2} +....+ A^{*r-1})} \tag{15}$$

Because the endogenous accounts are three, we can fix $r = 3$. Then we can rewrite equation (15) as:

$$\mathbf{t = (I- A^{*3})^{-1} (I + A^* + A^{*2}) M_1 x} \tag{16}$$

Equation (16) can be written as:

$$\mathbf{t = M_3M_2M_1 x} \tag{17}$$

$\mathbf{M_2}$ explicitly recognizes the interconnected character of the economic system. In fact, it captures the effects that an exogenous injection into an account of one block (for example, into one production activity) transmits to the endogenous accounts of an other block (for example, on households) due to the circulation of income flows. We can refer to $\mathbf{M_2}$ as open-loop multiplier. The open loop effects are measured by the impact of an exogenous shock from any vector $\mathbf{x_j}$ over the elements of the other $\mathbf{t_k}$ accounts with $j \neq k$. This matrix "explains why and how the stimulation of one part of the system has repercussions for all others" (Pyatt, Round, 2006, p. 239).

Finally:

$$\mathbf{M_3 = (I-A^{*3})^{-1} = \begin{vmatrix} {}_3M_{11} & 0 & 0 \\ 0 & {}_3M_{22} & 0 \\ 0 & 0 & {}_3M_{33} \end{vmatrix}} \tag{18}$$

where:

$$\mathbf{{}_3M_{11} = (I- A^*_{13} A^*_{32} A^*_{21})^{-1} = [I - (I- A_{11})^{-1}A_{13} (I- A_{33})^{-1}A_{32} A_{21}]^{-1}} \tag{19}$$

$$\mathbf{{}_3M_{22} = (I- A^*_{21} A^*_{13} A^*_{32})^{-1} = [I - A_{21}(I- A_{11})^{-1}A_{13} (I- A_{33})^{-1}A_{32}]^{-1}} \tag{20}$$

$$\mathbf{{}_3M_{33} = (I- A^*_{32} A^*_{21} A^*_{13})^{-1} = [I - (I- A_{33})^{-1}A_{32} A_{21}(I- A_{11})^{-1}A_{13}]^{-1}} \tag{21}$$

If we assume that $\mathbf{A_{33}= 0}$ equation (21) becomes

$$ {}_{3}\mathbf{M}_{33} = [\mathbf{I}- \mathbf{A}_{32}\, \mathbf{A}_{21}\, (\mathbf{I}- \mathbf{A}_{11})^{-1}\mathbf{A}_{13}]^{-1} \qquad (22) $$

$\mathbf{M_3}$ is the matrix of the closed loop multipliers and enlighten the circular structure of the system. Each element i (i = 1, 2, 3) of its diagonal blocks measures the multiplying impact of one exogenous shock in vector $\mathbf{x}_i$ on the endogenous account $\mathbf{t_i}$ at the end of the circular loop. It represents the "consequences of a change on **x** travelling around the entire system to reinforce the initial injection" (Pyatt, Round, p. 239)

If we focus our attention on the determination of the income distributed within the endogenous Private Institutions the corresponding $\mathbf{t_3}$ vector is given by:

$$ \mathbf{t}_3 = \mathbf{M}_{33}\, \mathbf{M}_{32}\, \mathbf{M}_{31}\, \mathbf{x} = \mathbf{M}_{31}\, \mathbf{x}_1 + \mathbf{M}_{32}\, \mathbf{x}_2 + \mathbf{M}_{33}\, \mathbf{x}_3 \qquad (23) $$

Where $\mathbf{M}_{31}$, $\mathbf{M}_{32}$, $\mathbf{M}_{33}$ can be expressed as:

$$ \mathbf{M}_{31} = {}_{3}\mathbf{M}_{33}\ {}_{2}\mathbf{M}_{31}\ {}_{1}\mathbf{M}_{11} \qquad (24) $$

$$ \mathbf{M}_{32} = {}_{3}\mathbf{M}_{33}\ {}_{2}\mathbf{M}_{32} \qquad (25) $$

$$ \mathbf{M}_{33} = {}_{3}\mathbf{M}_{33}\ {}_{1}\mathbf{M}_{33} \qquad (26) $$

Equation (23) allows us determining the total income of each group of the Private Institutions by the $\mathbf{M}_{31}$ $\mathbf{M}_{32}$ and $\mathbf{M}_{33}$ multipliers. The sum of the elements of the matrix $\mathbf{M}_{31}$ indicates the increase in the overall income of Private Institutions due to an exogenous injection of one unit in the income of each Activity account. The corresponding sums concerning $\mathbf{M}_{32}$ and $\mathbf{M}_{33}$ matrices indicate the increase in the overall income of Private Institutions due to an exogenous injection of one unit in the income of each Factor or each Private Institution. The column totals of these matrices are real income multipliers. Each of them, in fact, indicates by how much the overall income of Private Institutions would rise if the income of the corresponding elements in Activity, Factor or Private Institutions accounts would exogenously increase by one unit. Instead row totals indicate the multiplier effect on the income of any Private Institution in the case in which the income of each Activity Sector, each Factor or each Private Institution would increase by one unit.

The multiplier matrix **M** assumes a precise meaning with reference to a structural analysis of the income distribution of the Households. The elements of this matrix related to Private Institutions have the meaning, at a disaggregated level, of a Keynesian expenditure/income multiplier. Its value depends on the linkages between productive and spending structures (consumption expenditure, input-output relationships, value added distributed to different Households groups according to their ownership of the production Factors), and hence as a result of the mechanisms explicitly considered in the SAM model. Therefore it is a general framework for analysing the relationship between the distribution of income and the structure of the production.

The multiplier matrix $\mathbf{M}_{33}$, in particular, can be considered as a "structural" measure of inequality in the personal income distribution since it derives from the product of the components relating to Private Institutions in the $\mathbf{M}_1$ and $\mathbf{M}_3$ multipliers. It captures, in fact, the transfer effects (related to matrix $\mathbf{M}_1$) and the closed-loop effects (related to matrix $\mathbf{M}_3$) that involve only Private Institutions and the Activity Sector as in the input-output modelling. Considering our focus on income distribution of the private institutions, from equations 24-26, we can notice that the common element is matrix ${}_3\mathbf{M}_{33}$. Each element (${}_3\mathbf{M}_{ij}$) represents the income received by the *i*-group as a consequence of a change in the expenditure of disposable income of the *j*-group. Matrix ${}_3\mathbf{M}_{33}$ acquires then specific meaning of an income multiplier through the consumption expenditure as a result of a four-step "propagation" process. As also seen in Figure 2, the first step is represented by the matrix $\mathbf{A}_{13}$ of consumption coefficients with reference to disposable income of each of the Endogenous Private Institutions. The second step corresponds to that traditionally captured by the Leontief's inverse matrix transforming expenditure by sectors into intermediate output and determining the shares of the value added generated in the productive process. The third step, corresponding to the product of matrix $\mathbf{A}_{32}$ and matrix $\mathbf{A}_{21}$, determines the value added received by the Endogenous Private Institutions in connection with their ownership of the production Factors. The fourth step, finally, given by $(\mathbf{I}\text{-}\mathbf{A}_{33})^{-1}$ corresponds to the redistribution of income between Endogenous Institutions. The income thus produced, distributed a redistributed, turns into new levels of expenditures for consumption and the process occurs again until an equilibrium position is achieved.

3. The decomposition of the "accounting multipliers" M: a development

Following Pyatt and Round (2006, p. 240) it is possible to examine in a "microscopic detail" each element m_{ij} corresponding to the crossing between the row element *i* and the column element *j* of the global accounting multiplier matrix **M**. In this way it is possible to better analysing the impact on the account *i* of any exogenous injection into the account *j*. The m_{ij} element of the matrix **M** can be expressed as:

$$m_{ij} = \mathbf{d'}_i\, \mathbf{M}\, \mathbf{d}_j = \mathbf{d'}_i\, \mathbf{M_3\, M_2\, M_1}\, \mathbf{d}_j = \mathbf{i'}\ \hat{\mathbf{r}}\mathbf{A}\hat{\mathbf{s}}\, \mathbf{i} \qquad (27)$$

where $\mathbf{d'}_i$ and $\mathbf{d}_j$ are vectors in which respectively the *i*th element and the *j*th are equal to 1 and all others elements are equal to 0 (Pyatt, Round, 2006, p. 240). In vector **i** all elements are equal to 1. The matrix **A** and the vectors **r'** and **s** are defined as:

$$\mathbf{r'}=\mathbf{d'}_i\, \mathbf{M}_3 \quad \mathbf{A} = \mathbf{M}_2 \quad \text{and } \mathbf{s} = \mathbf{M}_1\mathbf{d}_j \qquad (28)$$

The equation (27) indicates that each m_{ij} must be equal to the sum of all elements of an $\hat{\mathbf{r}}\mathbf{A}\hat{\mathbf{s}}$ type transformation of the matrix $\mathbf{M}_2$ where, as we can see from (28), $\hat{\mathbf{r}}$ is a diagonal matrix formed by the *i*th row of the $\mathbf{M}_3$ multiplier, and $\hat{\mathbf{s}}$ is a diagonal matrix formed by the *j*th column of $\mathbf{M}_1$ (Pyatt, Round, 2006, p. 240). In this way it is possible to capture the across, the direct and indirect effects, from account *j* to account *i* ($i \neq j$) at a very disaggregated level. A complete accounting for m_{ij} can be constructed for any *i* and *j* from three elements i.e. the *i*th row of the matrix $\mathbf{M}_3 = (\mathbf{I}\text{-}\mathbf{A}^{*3})^{-1}$, the entire matrix $\mathbf{M}_2 = (\mathbf{I} + \mathbf{A}^* + \mathbf{A}^{*2})$ and the *j*th column of the matrix $\mathbf{M}_1 = (\mathbf{I}\text{-}\mathbf{A}_0)^{-1}$. The matrix $\hat{\mathbf{s}}$ shows how the consequences of a particular injection into the account *j* "will be amplified as a result of transfer effects within the category of accounts in which the initial stimulus arises" (Pyatt, Round, 2006, p. 240). The matrix $\mathbf{A}=\mathbf{M}_2$ explains how these initial effects will spread across the accounts belonging to other categories, that is the so called open loop effect. Finally $\hat{\mathbf{r}}$ "quantifies the consequences for account *i* of the circulation around the entire system of the *stimuli* generated *via* the first two mechanisms" (Pyatt, Round, 2006, p. 241).

All three mechanisms are important for diagnostic reasons since they allow us to account for m_{ij} in microscopic detail. The point can be better illustrated with reference to some specific examples where i is a particular Households group ($i \in$ H) and j is alternatively a particular sector of activity ($j \in$ A) or a particular factor of production ($j \in$ F). Recalling that both $\mathbf{M_1}$ and $\mathbf{M_3}$ are block diagonal matrices, it follows from (17) that the element m_{ij} of $\mathbf{M}$ will now be written as:

$$\mathbf{M_{H,A}} = {}_3\mathbf{M_{H,H}}\ {}_2\mathbf{M_{H,A}}\ {}_1\mathbf{M_{A,A}} \tag{29}$$

If the row i belongs to an household group H and the column j belongs to an activity sector A

$$m_{ij} = (\mathbf{d'}_i\ {}_3\mathbf{M_{H,H}})\ {}_2\mathbf{M_{H,A}} \quad ({}_1\mathbf{M_{A,A}}\ \mathbf{d}_j) \tag{30}$$

or if the column j is one of the production factor F

$$\mathbf{M_{H,F}} = {}_3\mathbf{M_{H,H}}\ {}_2\mathbf{M_{H,F}}\ {}_1\mathbf{M_{F,F}} \tag{31}$$

and therefore $$m_{ij} = (\mathbf{d'}_i\ {}_3\mathbf{M_{H,H}})\ {}_2\mathbf{M_{H,F}} \quad \mathbf{I} \tag{32}$$

equations (30) e (32) can be written in the form $\mathbf{i'(\hat{r}A\hat{s})i}$ where alternatively:

$$\mathbf{r'} = \mathbf{d'}_i\ {}_3\mathbf{M_{H,H}} \quad \mathbf{A} = {}_2\mathbf{M_{H,A}} \qquad \mathbf{s} = {}_1\mathbf{M_{A,A}}\ \mathbf{d}j \tag{33}$$
$$\mathbf{r'} = \mathbf{d'}_i\ {}_3\mathbf{M_{H,H}} \quad \mathbf{A} = {}_2\mathbf{M_{H,F}} \qquad \mathbf{s} = {}_1\mathbf{M_{F,F}}\ \mathbf{d}j \tag{34}$$

From (33) and (34) it results that the cell m_{ij} is equal to the sum of all elements of a $\mathbf{\hat{r}A\hat{s}}$ type transform of the matrix $\mathbf{M_2}$ in which $\mathbf{r'}$ is the i row of the block matrix ${}_3\mathbf{M_{H,H}}$; $\mathbf{A}$ is equal, alternatively, to the block matrix ${}_2\mathbf{M_{H,A}}$ or ${}_2\mathbf{M_{H,F}}$; $\mathbf{s}$ is the j column of the block matrix ${}_1\mathbf{M_{A,A}}$ or alternatively of ${}_1\mathbf{M_{F,F}} = \mathbf{I}$. This decomposition allows showing in a clear way the consequences of an exogenous injection in the jth Activity/Factor on the ith Household. The ${}_2\mathbf{M_{H,A}}$ and ${}_2\mathbf{M_{H,F}}$ are the matrices of the across effects and they explain how the original injection into the Activities/Factors accounts has repercussions in the Households account. These matrices have been bordered by the two vectors $\mathbf{r'}$ and $\mathbf{s}$. These are respectively: 1) in the first case the ith row of the matrix ${}_3\mathbf{M_{H,H}}$ and the jth column of the matrix ${}_1\mathbf{M_{A,A}}$; 2) in the second case the ith row of the matrix ${}_3\mathbf{M_{H,H}}$ and the jth column of the matrix ${}_1\mathbf{M_{F,F}}$.

An unit injection toward the jth Activity/Factor is directly translated by the '**A**' part of the $\mathbf{\hat{r}A\hat{s}}$ transform *i.e.* by the matrix ${}_2\mathbf{M_{H,A}}$ or ${}_2\mathbf{M_{H,F}}$ into increments of the incomes for the endogenous Institutions. The multiplier transfer effects within the Activities account are captured by the matrix ${}_1\mathbf{M_{A,A}}$. In the case of Factors there are no multiplier transfer effects within the account, because the multiplier ${}_1\mathbf{M_{F,F}}$ is equal to $\mathbf{I}$. Finally, the transmission of these increments right around the system - the complete circular flow - generates the impacts on the Household i that are captured by the row i of the multiplier matrix ${}_3\mathbf{M_{H,H}}$

4. The decomposition of the accounting multipliers for the Italian economic system

The meaning and the relevance of the multiplier approach in the use of the SAM as a simulation model can be illustrated with an application to the Italian economic system. The base SAM for Italy has been determined by the authors (Bottiroli Civardi, Chiappero Martinetti, Targetti Lenti, 1990) in a past research work, for the year 1984. The construction of the Italian SAM required an extensive processing of data drawn from different sources and the introduction of very simplifying hypothesis. This exercise must be considered mostly as an application to highlighting the potentiality of the approach, rather than a simulation bringing to unquestionable results.

The choice of number and of type of Institutions, and mostly the choice of the groups of the Institution Households is one of the more important steps of the SAM building process. Disaggregating of the factors and Household accounts are fundamental to any SAM. The choice of the single units, the estimate of data (often by survey), the inclusion or exclusion of some variables depends on the researcher goals. If the goal is mostly to build an analytical tool from which to obtain indicators of the labour market, of the employment and unemployment structure, the classification can be based on the "prevailing income" in the household. This kind of classification, suggested by the SNA93 and the SEC95, allows showing very well the links between primary distribution of income and structure of employment and/or of the production technology.

This taxonomy, however, mainly reproduces at the Households level the factorial distribution of income. If the groups are identified by the type of prevailing incomes, it could be difficult to distinguish between factorial and personal distribution of income, and to capture the linkages between income distribution and consumption expenditure. The aim, which is a distinguish feature of the SAM, to capture the link between factorial and Institutional/personal income distribution suggests grouping the Households according to their level of income.

Following a Keynesian approach, the H socio-economic groups must be chosen so that the propensities to consume are quite homogenous inside of them, but different group from group. Econometric analysis, in Italy, but also in other countries, shows that not only the level but also the composition of consumption is strongly affected by the amount of disposable income of the Households. A classification of the Households by class of income (deciles of population) has been therefore, chosen in this application because it seems the more suitable to assess the effects of any exogenous injection (a change in fiscal policies, for instance) on the income distribution vector of the Households groups. These policies are generally calibrated on the level of total income of each group of Households and not on the source (from the factorial side) of the income.

The data, which allowed estimating the personal income distribution of the Institution Households, were drawn by the biannual Survey of the Bank of Italy. The Activities were classified in seven branches (Agriculture, Industry, Trade, Transports, Credit and Insurance, Public Administration, Other Services). Endogenous Institutions has been separated in 10 groups of Households (deciles of population) according to their level of disposable income and in one account grouping all the Companies. The estimation of the matrix $\mathbf{S}_{3,2}$ (Figure 1) related to the ownership of factors by the Endogenous Institutions has been obtained starting from the census survey of 1981. The Factor accounts have been disaggregated by the authors into five categories (Employed Labour, Self-employed Labour, Capital in

Productive Activities, Capital in Housing, Financial Capital). The estimation of matrix $\mathbf{S_{1,3}}$ related to the consumption of different categories of commodities by each Household group has been obtained according to the correspondence between household budget and the input-output categories, i.e. starting from the so called "bridge matrix". All the other accounts of the base SAM were aggregated into the vector of Exogenous Institutions.

The global multiplier **M** is provided by the detailed set out in Table 1. The calculus of the multiplier blocks $\mathbf{M_{H,A}}$ and $\mathbf{M_{H,F}}$ of matrix **M** allows quantifying the effects on the income of the Institution Households from an exogenous injection of income alternatively directed to all the Activities or to all the Factors. One unit of income exogenously directed toward the Activities account generates an increase equal to 9,567 on the Households sector. One unit of income exogenously directed toward the Factors account, instead, generates an increase equal to 7,797. These values are the result of all the mechanisms induced by the closed-loop nature of the process.

A reading by row of values of the two multiplier blocks $\mathbf{M_{H,A}}$ and $\mathbf{M_{H,F}}$ shows the different ability of various Activities or Factors to generate income for each decile. These are considerably differentiated over the various deciles. As expected, row value totals show monotonically growing values with rather high differences between deciles, and multipliers of the last decile are always much higher than the first decile multipliers.

A reading by column shows the contribution of each Activity or Factor to the rise of the Households income. The contribution of each Activity (column total) is fairly differentiated. Credit and Insurance, followed by Public Administration and Transports, are the Activities showing the higher income multiplier for the Institution Households. These are sectors in which the share of the value added going to labour, and hence affecting directly the Institution Households, is larger. The effects produced on each decile by an increase attributed to Factors are only slightly less differentiated. Yet, despite the greater value it displays, Employed Labour does not seem to play a dominant multiplying effect as compared to Self-employment and Financial Capital.

Matrix $\mathbf{M_{H,H}}$ (Table 1) diagonal elements represent the income multiplier within each Endogenous Institution (deciles of population and Companies) generated by an additional unit of disposable income exogenously attributed to the group itself. With reference to Households, they are obviously all higher than one, and show a monotonically growing trend from the first to the last decile. This means that, as a consequence of an exogenous injection of additional income equally done, the final effect within the poorest group is always weaker than within the richest. The poorest decile has a lower ability to generate income for themselves than for generating income for the Institution Households as a whole. The global multiplier effect is equal to 17,997.

The total row values of $\mathbf{M_{H,H}}$ reflect the degree of inequality in the income distribution over Endogenous Institutions which can be considered structural, i.e. the distribution related to the values of the coefficients of expenditure, of intermediate production, of value added distribution and redistribution among them. All these values show a monotonically upward trend. The value for the first decile is rather small and it indicates the reduced potential of the system to distribute income to the poorest, while the multiplier effect in favour of the last decile appears to be particularly strong. The global multiplier $\mathbf{M_{H,H}}$ for the Institution Households (equal to 17,717) is quite completely determined (98,7%) by $_{\mathbf{3}}\mathbf{M_{H,H}}$ (Table 4), that is by the closed-loop nature of the process.

It is possible, then, as a first application, to examine in a "microscopic detail" the element m_{ij} of the global multiplier matrix **M** in equations (30) and (32) derived starting from our SAM for Italy when i is alternatively: 1) $\mathbf{F_1}$ (Employed Labour) and $\mathbf{F_3}$ (Capital in Productive Activities), with reference to the Factors account; 2) $\mathbf{A_1}$ (Agriculture) and $\mathbf{A_6}$ (Public Administration), with reference to the Activities account. The Household group j is alternatively the first and the last decile ($\mathbf{H_1}$ and $\mathbf{H_{10}}$). Tables 5 and 6 provide empirical estimates of the matrices on the right-hand side in equations (33) and (34).

First of all, we calculate $\hat{\mathbf{r}}\mathbf{A}\hat{\mathbf{s}}$ type transform in which **r'** is the i row of $_3\mathbf{M_{H.H}}$, **A** is equal to $_2\mathbf{M_{H,F}}$ and **s** is the j column of $_1\mathbf{M_{F,F}} = \mathbf{I}$. This means that we must border the matrix $_2\mathbf{M_{H,F}}$ by the row corresponding to the first/last decile in the matrix $_3\mathbf{M_{H,H}}$ and by the column corresponding to the first/third factor in the matrix $_1\mathbf{M_{F,F}}$. Since $_2\mathbf{M_{H,F}}$ is a 10 x 5 matrix, the result (32) provides a disaggregation of m_{ij} into 50 components for each i and j.

When alternatively i is $\mathbf{H_1}$ or $\mathbf{H_{10}}$ and j is $\mathbf{F_1}$ or $\mathbf{F_3}$ the corresponding m_{ij} are: $m_{H1,F1}$ =0,0269, $m_{H10,F1}$ = 0,4460 while $m_{H1,F3}$ = 0,0106 and $m_{H10,F3}$ = 0,3709. These values show some significant differences in the transmission effects from factors to Households (Table 5). From factor $\mathbf{F_1}$ to $\mathbf{H_1}$ the direct effect represents only the 56,81% of the total effect on all Households. This percentage decreases to 33,90% when the factor is $\mathbf{F_3}$. On the opposite side, when the i account is the 10th decile the direct effect rises respectively to 62,22% and to 79,15%.

In the same way if we want to analyze the element m_{ij} when alternatively i is $\mathbf{H_1}$ or $\mathbf{H_{10}}$ and j is $\mathbf{A_1}$ (Agriculture) or $\mathbf{A_6}$ (Public Administration), we must calculate a $\hat{\mathbf{r}}\mathbf{A}\hat{\mathbf{s}}$ type transform in which **r'** is the row i (corresponding to the first or the last decile) of $_3\mathbf{M_{H,H}}$ (Table 4), **A** is equal to $_2\mathbf{M_{H,A}}$ (Table 3) and **s** is the column j (corresponding to Agriculture or Public Administration) of $_1\mathbf{M_{A,A}}$ (Table 2). Therefore we must border the matrix $_2\mathbf{M_{H,A}}$ by the row corresponding to the first or the last decile in the matrix $_3\mathbf{M_{H,H}}$ and by the column corresponding to the first and third Activity in the matrix $_1\mathbf{M_{A,A}}$. Since $_2\mathbf{M_{H,A}}$ is a 10 x 7 matrix, the result (30) provides a disaggregation of m_{ij} into 70 components for each i and j.

An unit injection in Agriculture or, alternatively, in the Public Administration, generates multiplier effects on the various sectors of activity the magnitude of which can be read-off from the relevant column of the input-output inverse $\mathbf{(I\text{-}A_{11})}^{-1} = {}_1\mathbf{M_{A,A}}$ as reported in Table 2. The matrix $_2\mathbf{M_{H,A}}$ in Table 3, i.e. the **A** part of the $\hat{\mathbf{r}}\mathbf{A}\hat{\mathbf{s}}$, translates these effects into increments of income for the various Households deciles. Finally, the transmission of these increments right around the system -the complete circular flow- generates the implications for the first/last Household decile. These last effects are captured by the row of multiplier $_3\mathbf{M_{H,H}}$ that corresponds to account $\mathbf{H_1}$ or $\mathbf{H_{10}}$.

Table 6 represent for each sector $\mathbf{A_1}$ and $\mathbf{A_6}$ the capacity to stimulate (directly or indirectly) the income of $\mathbf{H_1}$ or $\mathbf{H_{10}}$. In this case: $m_{H1,A1}$ = 0,0154, $m_{H10,A1}$ = 0,3345 while $m_{H1,A6}$ = 0,0241, $m_{H10,A6}$ = 0,4183 showing significant differences in the transmission effects from Activities to Households. In rows 1st and 7th are showed the values of the 1st and the 6th column ($\mathbf{A_1}$ and $\mathbf{A_6}$) of the matrix $\hat{\mathbf{r}}_\mathbf{i}\mathbf{A}\hat{\mathbf{s}}_\mathbf{j}$ when the household group is $\mathbf{H_1}$; while in the 4th and 10th rows are showed the values of the 1st and of the 6th column of the matrix $\hat{\mathbf{r}}_\mathbf{i}\mathbf{A}\hat{\mathbf{s}}_\mathbf{j}$ when the household group is $\mathbf{H_{10}}$. We observe that the total direct effect is alternatively 0,0059 from $\mathbf{A_1}$ to $\mathbf{H_1}$ (38,53% of the total) and 0,0096 (56,20% of the total) from $\mathbf{A_6}$ to $\mathbf{H_1}$. The share of this direct effect rises when the household group is $\mathbf{H_{10}}$: it is equal to 0,1880 (the 56,20%) of the total from $\mathbf{A_1}$ to $\mathbf{H_{10}}$ and equal to 0,1780 (the 42,57% of the total) from $\mathbf{A_6}$ to $\mathbf{H_{10}}$.

The difference between the first row total (0,115) and the direct effect is equal to 0,056 (94,9%). This is a measure of the direct effect from $\mathbf{A_1}$ as the consequence of the level of activation of incomes of other households. This effect for $\mathbf{H_{10}}$ (4th row) is equal to 0,0760 (40,4%). In the case of $\mathbf{A_6}$, instead, the effect on $\mathbf{H_1}$ due to the activation of incomes of the other households is 0,0073 (43,2%) while on $\mathbf{H_{10}}$ is 0,1057 (37,3%). Therefore, for poorer households the capacity of $\mathbf{A_1}$ and $\mathbf{A_6}$ to activate income through the income of other households is higher, in relative terms, than that of the richer ones. This is particularly true for $\mathbf{A_1}$.

The direct linkages of the first decile, both with the Agriculture sector and the Public Administration, in the case of Italy, result weaker than the linkages with the last decile. Moreover the direct effect toward $\mathbf{H_{10}}$ from $\mathbf{A_1}$ is higher than from $\mathbf{A_6}$. The opposite happens when we consider the group $\mathbf{H_1}$. These values, however, reflect the inequality in the property of Factors that becomes inequality in the personal income distribution.

The 2nd and 5th rows (and the 8th and 11th) of table 6 represent the values of the other columns of the matrix $\mathbf{\hat{r}_i A \hat{s}_j}$ summed up, i.e. they represent the indirect effects from other Activities to each of the two Households decile considered. This impact is equal to 0,0021 and 0,0527 from $\mathbf{A_1}$ alternatively to $\mathbf{H_1}$ and $\mathbf{H_{10}}$ and equal to 0,0042 and 0,0948 from $\mathbf{A_6}$ to alternatively $\mathbf{H_1}$ and $\mathbf{H_{10}}$. It operates also through the incomes of other Households (equal to 0,0018 and 0,0730 from $\mathbf{A_1}$ to alternatively $\mathbf{H_1}$ and $\mathbf{H_{10}}$ and equal to 0,0030 and 0,1336 from $\mathbf{A_6}$ to alternatively $\mathbf{H_1}$ and $\mathbf{H_{10}}$).

In the case of our exercise these indirect effects are always significantly lower than the direct ones. In some other examples, as for the Indonesian case, it happens exactly the opposite: the direct effect from “food processing” to “small scale farm household” was not the most important (Pyatt, Round, 2006, p. 255). “More powerful linkages” were “generated by the increased intermediate demand for food crops …. as a result of the stimulation of the food processing sector. This derived demand evidently creates significant extra income for all the household groups … which, in turn, generate extra income for “small scale farm household” (Pyatt, Round, 2006, p. 242).

5. Concluding remarks

The decomposition of the accounting multiplier matrix allows isolating the value of different multipliers and better assessing the linkages between Households income group, property of factors and sectors of activities. It allows capturing direct and indirect effects of an exogenous injection in Factors or in Activities Account on each one of the endogenous Institutions. This kind of analysis can drive toward a better understanding of the relationships between inequality, poverty and alternative policies.

The analysis done bring us to affirm that in an economic system, like that here analyzed, and probably in any market economy, the benefits produced by an increase in disposable income, initially equally earned by all the Households groups, propagate through ways of spending so as to increasingly favour the upper-middle deciles, and particularly the last ones. The labour market, the ownership of factors, the technological features of the production process stays “behind” the level of the inequality. The closed-loop process is something strictly interwoven with the operating of the market and can be considered a special feature of every system.

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Appendix

Table 1 - GLOBAL MULTIPLIER MATRIX M

		ACTIVITIES							FACTORS				
		A_1	A_2	A_3	A_4	A_5	A_6	A_7	F_1	F_2	F_3	F_4	F_5
A_1	AC	1,313	0,147	0,094	0,122	0,121	0,120	0,115	0,110	0,098	0,063	0,093	0,106
A_2	T	0,914	2,153	0,953	1,319	1,239	1,160	0,996	0,995	0,892	0,586	0,830	0,979
A_3	I	0,195	0,170	1,238	0,248	0,238	0,238	0,211	0,248	0,220	0,144	0,207	0,242
A_4	V	0,135	0,135	0,167	1,296	0,185	0,173	0,155	0,170	0,183	0,122	0,145	0,198
A_5	I	0,101	0,114	0,115	0,147	1,105	0,218	0,101	0,078	0,071	0,047	0,065	0,078
A_6	TI	0,005	0,006	0,008	0,009	0,008	1,009	0,007	0,008	0,007	0,005	0,007	0,008
A_7	ES	0,250	0,253	0,351	0,383	0,456	0,434	1,338	0,371	0,331	0,217	0,310	0,365
Total Activities		**2,912**	**2,978**	**2,927**	**3,523**	**3,352**	**3,352**	**2,923**	**1,981**	**1,802**	**1,184**	**1,657**	**1,976**
%		*51,62*	*56,89*	*48,43*	*47,87*	*46,59*	*48,07*	*47,85*	*33,29*	*32,92*	*26,49*	*31,60*	*33,29*
F_1	FA	0,564	0,579	0,615	0,910	0,772	1,115	0,643	1,419	0,386	0,254	0,351	0,423
F_2	CT	0,254	0,126	0,332	0,189	0,139	0,134	0,198	0,126	1,113	0,074	0,105	0,124
F_3	O	0,394	0,272	0,428	0,585	0,391	0,294	0,361	0,235	0,219	1,144	0,197	0,240
F_4	R	0,054	0,054	0,075	0,082	0,098	0,097	0,286	0,080	0,071	0,046	1,066	0,078
F_5	S	0,039	0,045	0,045	0,058	0,433	0,085	0,040	0,031	0,028	0,018	0,025	1,031
Total Fattors		**1,305**	**1,076**	**1,495**	**1,823**	**1,832**	**1,725**	**1,528**	**1,891**	**1,817**	**1,537**	**1,746**	**1,896**
%		*23,14*	*20,55*	*24,73*	*24,77*	*25,46*	*24,74*	*25,01*	*31,77*	*33,20*	*34,40*	*33,30*	*31,94*
H_1	IN	0,015	0,014	0,018	0,022	0,026	0,024	0,023	0,027	0,023	0,011	0,042	0,030
H_2	S	0,041	0,036	0,048	0,057	0,058	0,061	0,052	0,070	0,066	0,030	0,068	0,058
H_3	T	0,057	0,049	0,066	0,079	0,081	0,084	0,071	0,096	0,087	0,046	0,089	0,080
H_4	I	0,070	0,060	0,080	0,097	0,098	0,102	0,084	0,117	0,105	0,057	0,098	0,096
H_5	T	0,081	0,072	0,092	0,116	0,120	0,126	0,098	0,146	0,108	0,062	0,111	0,121
H_6	U	0,096	0,084	0,110	0,135	0,137	0,145	0,116	0,167	0,136	0,075	0,134	0,133
H_7	T	0,140	0,120	0,159	0,201	0,199	0,199	0,164	0,224	0,176	0,155	0,176	0,195
H_8	I	0,114	0,108	0,127	0,176	0,187	0,192	0,145	0,223	0,093	0,108	0,168	0,193
H_9	O	0,180	0,151	0,208	0,247	0,268	0,252	0,208	0,283	0,284	0,158	0,222	0,300
H_{10}	NS	0,334	0,266	0,388	0,447	0,494	0,418	0,373	0,446	0,552	0,371	0,392	0,591
Total Households		**1,129**	**0,959**	**1,296**	**1,578**	**1,668**	**1,602**	**1,335**	**1,799**	**1,630**	**1,072**	**1,500**	**1,796**
%		*20,00*	*18,31*	*21,44*	*21,44*	*23,18*	*22,98*	*21,85*	*30,23*	*29,79*	*24,00*	*28,61*	*30,26*
Companies		0,295	0,223	0,327	0,436	0,344	0,294	0,323	0,281	0,224	0,675	0,339	0,268
TOTAL		**5,642**	**5,235**	**6,045**	**7,359**	**7,195**	**6,974**	**6,108**	**5,951**	**5,474**	**4,468**	**5,241**	**5,936**
H_{10}/H_1		***21,71***	***19,33***	***21,50***	***20,55***	***19,24***	***17,32***	***16,00***	***16,59***	***24,22***	***34,91***	***9,33***	***19,77***

Table 1 (continued) **- GLOBAL MULTIPLIER MATRIX M**

		INSTITUTIONS											TOTAL
		H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	Comp.	
A_1	AC	0,136	0,142	0,140	0,131	0,115	0,123	0,114	0,103	0,094	0,096	0,023	3,720
A_2	T	1,003	1,070	1,094	1,046	0,944	1,016	1,106	0,992	0,908	0,962	0,220	23,376
A_3	I	0,263	0,279	0,282	0,265	0,241	0,256	0,267	0,254	0,230	0,228	0,054	6,219
A_4	V	0,111	0,120	0,120	0,130	0,118	0,127	0,138	0,130	0,119	0,327	0,033	4,537
A_5	I	0,076	0,080	0,081	0,078	0,071	0,077	0,083	0,077	0,071	0,083	0,017	3,033
A_6	TI	0,008	0,010	0,008	0,009	0,007	0,009	0,008	0,008	0,007	0,008	0,002	1,169
A_7	ES	0,405	0,415	0,408	0,379	0,342	0,366	0,391	0,385	0,371	0,347	0,080	8,948
Total Activities		**2,001**	**2,115**	**2,133**	**2,037**	**1,838**	**1,973**	**2,107**	**1,950**	**1,800**	**2,051**	**0,429**	**51,002**
%		*40,98*	*41,40*	*41,55*	*41,11*	*40,18*	*40,83*	*39,22*	*38,15*	*39,80*	*40,28*	*20,59*	*41,36*
F_1	FA	0,414	0,438	0,440	0,422	0,381	0,410	0,437	0,407	0,377	0,459	0,090	12,306
F_2	CT	0,132	0,139	0,139	0,132	0,119	0,127	0,133	0,125	0,116	0,125	0,027	4,228
F_3	O	0,231	0,243	0,244	0,235	0,212	0,227	0,241	0,226	0,209	0,269	0,050	7,146
F_4	R	0,087	0,089	0,087	0,081	0,073	0,078	0,084	0,083	0,079	0,074	0,017	2,919
F_5	S	0,030	0,031	0,032	0,030	0,028	0,030	0,032	0,030	0,028	0,032	0,007	2,189
Total Factors		**0,893**	**0,940**	**0,942**	**0,901**	**0,814**	**0,872**	**0,927**	**0,871**	**0,808**	**0,959**	**0,191**	**28,789**
%		*18,29*	*18,40*	*18,36*	*18,18*	*17,78*	*18,05*	*17,25*	*17,05*	*17,87*	*18,84*	*9,16*	*23,35*
H_1	IN	1,012	0,013	0,013	0,012	0,011	0,012	0,013	0,012	0,011	0,012	0,004	1,397
H_2	S	0,030	1,031	0,031	0,030	0,027	0,029	0,031	0,029	0,027	0,031	0,007	1,950
H_3	T	0,041	0,043	1,043	0,041	0,037	0,040	0,043	0,041	0,037	0,043	0,014	2,306
H_4	I	0,049	0,052	0,052	1,050	0,045	0,048	0,052	0,049	0,045	0,052	0,017	2,575
H_5	T	0,058	0,061	0,061	0,058	1,053	0,057	0,062	0,059	0,052	0,062	0,022	2,859
H_6	U	0,068	0,072	0,072	0,069	0,062	1,067	0,073	0,069	0,062	0,073	0,025	3,178
H_7	T	0,099	0,105	0,106	0,101	0,092	0,099	1,133	0,129	0,091	0,108	0,145	4,315
H_8	I	0,084	0,089	0,089	0,085	0,077	0,083	0,091	1,086	0,077	0,092	0,032	3,719
H_9	O	0,124	0,131	0,131	0,125	0,113	0,121	0,133	0,126	1,112	0,132	0,045	5,053
H_{10}	NS	0,223	0,235	0,236	0,225	0,204	0,218	0,238	0,225	0,202	1,238	0,076	8,392
Total Households		**1,787**	**1,831**	**1,833**	**1,797**	**1,721**	**1,773**	**1,871**	**1,824**	**1,716**	**1,844**	**0,386**	**35,744**
%		*36,59*	*35,83*	*35,71*	*36,27*	*37,61*	*36,68*	*34,83*	*35,69*	*37,93*	*36,22*	*18,50*	*28,99*
Companies		0,202	0,224	0,225	0,220	0,202	0,215	0,467	0,466	0,199	0,238	1,079	7,764
TOTAL		**4,883**	**5,110**	**5,134**	**4,954**	**4,575**	**4,833**	**5,372**	**5,112**	**4,523**	**5,091**	**2,085**	**123,299**
H_{10}/H_1		***0,22***	***18,67***	***18,72***	***18,80***	***18,80***	***18,80***	***18,89***	***18,80***	***18,51***	***99,95***	***21,16***	***6,01***

Table 2 - MULTIPLIER MATRIX M_1

	ACTIVITIES							FACTORS				
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	F_1	F_2	F_3	F_4	F_5
A_1	1,245	0,089	0,016	0,027	0,021	0,022	0,034	0	0	0	0	0
A_2	0,293	1,625	0,241	0,450	0,324	0,275	0,260	0	0	0	0	0
A_3	0,041	0,039	1,062	0,032	0,011	0,017	0,028	0	0	0	0	0
A_4	0,018	0,039	0,032	1,136	0,012	0,018	0,021	0	0	0	0	0
A_5	0,051	0,072	0,058	0,078	1,033	0,148	0,043	0	0	0	0	0
A_6	0,000	0,002	0,002	0,002	0,000	1,002	0,001	0	0	0	0	0
A_7	0,019	0,056	0,086	0,060	0,115	0,104	1,064	0	0	0	0	0
Total Activities	**1,668**	**1,922**	**1,498**	**1,784**	**1,516**	**1,587**	**1,451**	**0**	**0**	**0**	**0**	**0**
F_1	0	0	0	0	0	0	0	*1*	0	*0*	0	0
F_2	0	0	0	0	0	0	0	*0*	1	*0*	0	0
F_3	0	0	0	0	0	0	0	*0*	0	*1*	0	0
F_4	0	0	0	0	0	0	0	*0*	0	*0*	1	0
F_5	0	0	0	0	0	0	0	*0*	0	*0*	0	1
Total Factors	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**1**	**1**	**1**
H_1	0	0	0	0	0	0	0	0	0	0	0	0
H_2	0	0	0	0	0	0	0	0	0	0	0	0
H_3	0	0	0	0	0	0	0	0	0	0	0	0
H_4	0	0	0	0	0	0	0	0	0	0	0	0
H_5	0	0	0	0	0	0	0	0	0	0	0	0
H_6	0	0	0	0	0	0	0	0	0	0	0	0
H_7	0	0	0	0	0	0	0	0	0	0	0	0
H_8	0	0	0	0	0	0	0	0	0	0	0	0
H_9	0	0	0	0	0	0	0	0	0	0	0	0
H_{10}	0	0	0	0	0	0	0	0	0	0	0	0
Total Households	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
Companies	0	0	0	0	0	0	0	0	0	0	0	0
TOTAL	**1,668**	**1,922**	**1,498**	**1,784**	**1,516**	**1,587**	**1,451**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**

Table 2 (continued) - **MULTIPLIER MATRIX M_1**

	INSTITUTIONS											TOTAL
	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	Comp.	
A_1	0	0	0	0	0	0	0	0	0	0	0	**1,454**
A_2	0	0	0	0	0	0	0	0	0	0	0	**3,469**
A_3	0	0	0	0	0	0	0	0	0	0	0	**1,230**
A_4	0	0	0	0	0	0	0	0	0	0	0	**1,275**
A_5	0	0	0	0	0	0	0	0	0	0	0	**1,485**
A_6	0	0	0	0	0	0	0	0	0	0	0	**1,011**
A_7	0	0	0	0	0	0	0	0	0	0	0	**1,504**
Total Activities	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**11,426**
F_1	0	0	0	0	0	0	0	0	0	0	0	**1,000**
F_2	0	0	0	0	0	0	0	0	0	0	0	**1,000**
F_3	0	0	0	0	0	0	0	0	0	0	0	**1,000**
F_4	0	0	0	0	0	0	0	0	0	0	0	**1,000**
F_5	0	0	0	0	0	0	0	0	0	0	0	**1,000**
Total Factors	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**5,000**
H_1	1,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	**1,002**
H_2	0,000	1,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	**1,002**
H_3	0,000	0,000	1,000	0,000	0,000	0,000	0,001	0,001	0,000	0,000	0,005	**1,009**
H_4	0,000	0,000	0,000	1,000	0,000	0,000	0,002	0,002	0,000	0,000	0,006	**1,011**
H_5	0,000	0,000	0,000	0,000	1,000	0,000	0,002	0,003	0,000	0,000	0,009	**1,016**
H_6	0,000	0,000	0,000	0,000	0,000	1,000	0,003	0,003	0,000	0,000	0,010	**1,018**
H_7	0,001	0,003	0,003	0,003	0,003	0,003	1,032	0,034	0,003	0,003	0,125	**1,215**
H_8	0,000	0,000	0,000	0,000	0,000	0,000	0,004	1,004	0,000	0,000	0,013	**1,023**
H_9	0,000	0,000	0,000	0,001	0,001	0,001	0,005	0,005	1,000	0,000	0,019	**1,032**
H_{10}	0,000	0,001	0,001	0,001	0,001	0,001	0,007	0,008	0,001	1,001	0,028	**1,048**
Total Households	**1,003**	**1,005**	**1,005**	**1,006**	**1,006**	**1,006**	**1,057**	**1,059**	**1,006**	**1,005**	**0,218**	**10,375**
Companies	0,012	0,024	0,025	0,028	0,029	0,029	0,270	0,280	0,027	0,025	1,038	**1,788**
TOTAL	**0,015**	**0,028**	**0,030**	**0,033**	**0,035**	**1,035**	**1,321**	**1,333**	**1,032**	**1,030**	**1,233**	**28,589**

Table 3 - MULTIPLIER MATRIX M_2

	ACTIVITIES							FACTORS				
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	F_1	F_2	F_3	F_4	F_5
A_1	1	0	0	0	0	0	0	0,063	0,054	0,034	0,053	0,058
A_2	0	1	0	0	0	0	0	0,565	0,497	0,327	0,470	0,546
A_3	0	0	1	0	0	0	0	0,141	0,122	0,080	0,118	0,135
A_4	0	0	0	1	0	0	0	0,091	0,110	0,074	0,079	0,118
A_5	0	0	0	0	1	0	0	0,044	0,040	0,026	0,037	0,044
A_6	0	0	0	0	0	1	0	0,004	0,004	0,003	0,004	0,004
A_7	0	0	0	0	0	0	1	0,211	0,184	0,120	0,176	0,204
Total Activities	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1**	**1**	**1**	**1**	**1**
F_1	0,172	0,174	0,203	0,364	0,277	0,612	0,238	1	0	0	0	0
F_2	0,127	0,02	0,213	0,048	0,001	0	0,083	0	1	0	0	0
F_3	0,165	0,059	0,199	0,283	0,122	0,023	0,138	0	0	1	0	0
F_4	0	0	0	0	0	0,004	0,214	0	0	0	1	0
F_5	0	0	0	0	0,392	0	0	0	0	0	0	1
Total Factors	**0,464**	**0,253**	**0,615**	**0,695**	**0,792**	**0,639**	**0,673**	**1**	**1**	**1**	**1**	**1**
H_1	0,005	0,003	0,006	0,007	0,012	0,009	0,012	0,015	0,012	0,004	0,032	0,018
H_2	0,014	0,009	0,019	0,020	0,024	0,026	0,024	0,041	0,039	0,013	0,043	0,028
H_3	0,020	0,012	0,026	0,029	0,034	0,035	0,032	0,055	0,050	0,022	0,055	0,039
H_4	0,024	0,015	0,032	0,036	0,041	0,043	0,037	0,069	0,060	0,027	0,057	0,047
H_5	0,027	0,018	0,035	0,043	0,053	0,055	0,043	0,089	0,055	0,027	0,063	0,064
H_6	0,032	0,021	0,043	0,050	0,057	0,062	0,051	0,100	0,074	0,035	0,078	0,065
H_7	0,049	0,029	0,063	0,077	0,085	0,080	0,071	0,127	0,087	0,096	0,094	0,097
H_8	0,035	0,028	0,043	0,067	0,088	0,086	0,063	0,138	0,016	0,057	0,097	0,108
H_9	0,063	0,036	0,086	0,090	0,124	0,100	0,089	0,160	0,171	0,084	0,119	0,176
H_{10}	0,122	0,060	0,167	0,166	0,236	0,144	0,159	0,224	0,349	0,237	0,207	0,368
Total Households	**0,390**	**0,231**	**0,521**	**0,585**	**0,752**	**0,640**	**0,582**	**1,018**	**0,914**	**0,602**	**0,846**	**1,011**
Companies	0	0	0	0	0	0	0	0	0	0	0	0
TOTAL	**1,854**	**1,484**	**2,136**	**2,280**	**2,544**	**2,279**	**2,255**	**3,137**	**2,926**	**2,266**	**2,781**	**3,121**
H_{10}/H_1	***25,91***	***19,52***	***26,38***	***23,40***	***20,13***	***15,21***	***13,32***	***14,90***	***29,99***	***66,70***	***6,43***	***20,40***

Table 3 (continued) - **MULTIPLIER MATRIX M_2**

	INSTITUTIONS											TOTAL
	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	Comp.	
A_1	0,088	0,092	0,089	0,083	0,072	0,077	0,061	0,053	0,051	0,045	0	1,974
A_2	0,569	0,612	0,635	0,607	0,547	0,590	0,624	0,536	0,513	0,497	0	9,136
A_3	0,156	0,165	0,168	0,156	0,143	0,150	0,148	0,141	0,132	0,112	0	3,067
A_4	0,031	0,035	0,036	0,049	0,045	0,048	0,051	0,048	0,046	0,241	0	2,104
A_5	0,041	0,044	0,045	0,043	0,040	0,044	0,045	0,042	0,040	0,046	0	1,620
A_6	0,004	0,006	0,004	0,005	0,004	0,005	0,004	0,004	0,004	0,004	0	1,065
A_7	0,244	0,244	0,238	0,215	0,194	0,207	0,212	0,216	0,224	0,174	0	4,064
Total Activities	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**0**	**23,030**
F_1	0,229	0,243	0,244	0,236	0,212	0,229	0,233	0,214	0,209	0,262	0	5,351
F_2	0,078	0,081	0,081	0,076	0,069	0,073	0,072	0,068	0,066	0,066	0	2,221
F_3	0,127	0,133	0,134	0,129	0,117	0,125	0,126	0,117	0,114	0,157	0	3,269
F_4	0,052	0,052	0,051	0,046	0,042	0,044	0,045	0,046	0,048	0,037	0	1,682
F_5	0,016	0,017	0,017	0,017	0,016	0,017	0,017	0,016	0,016	0,018	0	1,560
Total Factors	**0,502**	**0,527**	**0,528**	**0,504**	**0,455**	**0,488**	**0,494**	**0,462**	**0,452**	**0,540**	**0**	**14,083**
H_1	1,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1,135
H_2	0,000	1,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1,300
H_3	0,000	0,000	1,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1,409
H_4	0,000	0,000	0,000	1,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1,488
H_5	0,000	0,000	0,000	0,000	1,000	0,000	0,000	0,000	0,000	0,000	0,000	1,571
H_6	0,000	0,000	0,000	0,000	0,000	1,000	0,000	0,000	0,000	0,000	0,000	1,668
H_7	0,000	0,000	0,000	0,000	0,000	0,000	1,000	0,000	0,000	0,000	0,000	1,956
H_8	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1,000	0,000	0,000	0,000	1,826
H_9	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1,000	0,000	0,000	2,298
H_{10}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	1,000	0,000	3,440
Total Households	**1,000**	**1,000**	**1,000**	**10,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**0,000**	**18,091**
Companies	0,012	0,024	0,025	0,028	0,029	0,029	0,270	0,280	0,027	0,025	1,038	1,788
TOTAL	**2,648**	**2,750**	**2,768**	**2,690**	**2,528**	**2,639**	**2,909**	**2,782**	**2,491**	**2,685**	**1,038**	**56,991**
H_{10}/H_1	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	***3,03***

Table 4 - MULTIPLIER MATRIX M_3

	ACTIVITIES							FACTORS				
	A_1	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**	**A_7**	**F_1**	**F_2**	**F_3**	**F_4**	**F_5**
A_1	1,042	0,025	0,056	0,063	0,080	0,069	0,063	0	0	0	0	0
A_2	0,381	1,226	0,509	0,571	0,732	0,626	0,569	0	0	0	0	0
A_3	0,094	0,056	1,126	0,142	0,181	0,156	0,142	0	0	0	0	0
A_4	0,073	0,040	0,098	1,105	0,140	0,107	0,104	0	0	0	0	0
A_5	0,030	0,018	0,040	0,045	1,058	0,049	0,045	0	0	0	0	0
A_6	0,003	0,002	0,004	0,005	0,006	1,005	0,005	0	0	0	0	0
A_7	0,142	0,084	0,189	0,212	0,273	0,233	1,212	0	0	0	0	0
Total Activities	**1,765**	**1,451**	**2,022**	**2,143**	**2,470**	**2,246**	**2,139**	**0**	**0**	**0**	**0**	**0**
F_1	0	0	0	0	0	0	0	1,419	0,386	0,254	0,351	0,423
F_2	0	0	0	0	0	0	0	0,126	1,113	0,074	0,105	0,124
F_3	0	0	0	0	0	0	0	0,235	0,219	1,144	0,197	0,240
F_4	0	0	0	0	0	0	0	0,080	0,071	0,046	1,066	0,078
F_5	0	0	0	0	0	0	0	0,031	0,028	0,018	0,025	1,031
Total Factors	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**1,891**	**1,817**	**1,537**	**1,746**	**1,896**
H_1	0	0	0	0	0	0	0	0	0	0	0	0
H_2	0	0	0	0	0	0	0	0	0	0	0	0
H_3	0	0	0	0	0	0	0	0	0	0	0	0
H_4	0	0	0	0	0	0	0	0	0	0	0	0
H_5	0	0	0	0	0	0	0	0	0	0	0	0
H_6	0	0	0	0	0	0	0	0	0	0	0	0
H_7	0	0	0	0	0	0	0	0	0	0	0	0
H_8	0	0	0	0	0	0	0	0	0	0	0	0
H_9	0	0	0	0	0	0	0	0	0	0	0	0
H_{10}	0	0	0	0	0	0	0	0	0	0	0	0
Total Households	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
Companies	0	0	0	0	0	0	0	0	0	0	0	0
TOTAL	**1,765**	**1,451**	**2,022**	**2,143**	**2,470**	**2,246**	**2,139**	**1,891**	**1,817**	**1,537**	**1,746**	**1,896**

Table 4 (continued) **- MULTIPLIER MATRIX M_3**

	INSTITUTIONS											
	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	Comp.	TOTAL
A_1	0	0	0	0	0	0	0	0	0	0	0	1,397
A_2	0	0	0	0	0	0	0	0	0	0	0	4,613
A_3	0	0	0	0	0	0	0	0	0	0	0	1,897
A_4	0	0	0	0	0	0	0	0	0	0	0	1,667
A_5	0	0	0	0	0	0	0	0	0	0	0	1,285
A_6	0	0	0	0	0	0	0	0	0	0	0	1,029
A_7	0	0	0	0	0	0	0	0	0	0	0	2,346
Total Activities	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**14,234**
F_1	0	0	0	0	0	0	0	0	0	0	0	2,833
F_2	0	0	0	0	0	0	0	0	0	0	0	1,543
F_3	0	0	0	0	0	0	0	0	0	0	0	2,035
F_4	0	0	0	0	0	0	0	0	0	0	0	1,341
F_5	0	0	0	0	0	0	0	0	0	0	0	1,133
Total Factors	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**8,886**
H_1	1,012	0,013	0,013	0,012	0,011	0,012	0,012	0,011	0,011	0,012	0,000	1,117
H_2	0,030	1,031	0,031	0,030	0,027	0,029	0,029	0,027	0,027	0,031	0,000	1,291
H_3	0,040	0,042	1,043	0,041	0,037	0,039	0,040	0,037	0,036	0,043	0,000	1,398
H_4	0,049	0,051	0,051	1,049	0,044	0,048	0,048	0,045	0,044	0,052	0,000	1,482
H_5	0,057	0,060	0,061	0,058	1,052	0,056	0,057	0,053	0,052	0,061	0,000	1,568
H_6	0,068	0,071	0,071	0,068	0,061	1,066	0,067	0,062	0,061	0,072	0,000	1,667
H_7	0,097	0,102	0,102	0,098	0,088	0,094	1,096	0,089	0,087	0,104	0,000	1,958
H_8	0,084	0,088	0,088	0,085	0,076	0,082	0,083	1,078	0,076	0,091	0,000	1,831
H_9	0,123	0,130	0,130	0,124	0,112	0,120	0,122	0,113	1,111	0,131	0,000	2,216
H_{10}	0,222	0,233	0,234	0,223	0,202	0,216	0,218	0,204	0,200	1,237	0,000	3,190
Total Households	**1,782**	**1,822**	**1,824**	**1,786**	**1,719**	**1,762**	**1,771**	**1,720**	**1,706**	**1,835**	**0,000**	**17,717**
Companies	0,189	0199	0199	0,191	0,172	0,185	0,186	0,174	0,171	0,212	1,000	2,879
TOTAL	**1,971**	**2,021**	**2,023**	**1,977**	**1,882**	**1,946**	**1,958**	**1,895**	**1,876**	**2,046**	**1,000**	**43,716**

Table 5 - Decomposition of m_{ij}: i =H_1 and H_{10} and j = F_1 and F_3

Account j	Account i	m_{ij}	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	% Effect on Account i
F_1	H_1	0,0269	0,0153	0,0005	0,0007	0,0008	0,0010	0,0012	0,0015	0,0015	0,0017	0,0028	*56,81%*
F_1	H_{10}	0,4460	0,0034	0,0096	0,0130	0,0153	0,0179	0,0216	0,0277	0,0282	0,0319	0,2775	*62,22%*
F_3	H_1	0,0106	0,0036	0,0002	0,0003	0,0003	0,0003	0,0004	0,0011	0,0006	0,0009	0,0029	*33,90%*
F_3	H_{10}	0,3709	0,0008	0,0029	0,0051	0,0061	0,0054	0,0075	0,0210	0,0117	0,0168	0,2935	*79,15%*

Table 6 - Decomposition of m_{ij}: *i* =H_1 and H_{10} and *j* = A_1 and A_6

Account j	Account i	m_{ij}		H_1	H_2	H_3	H_4	H_5
A_1	H_1	**0,0154**	Direct Effect from Activity A_1	*0,0059*	0,0002	0,0003	0,0004	0,0004
			Indirect Effect from other Activities	*0,0021*	0,0001	0,0001	0,0001	0,0001
			Total Effect from Activity A_1	***0,0081***	**0,0003**	**0,0004**	**0,0005**	**0,0005**
A_1	H_{10}	**0,3345**	Direct Effect from Activity A_1	0,0013	0,0041	0,0057	0,0067	0,0067
			Indirect Effect from other Activities	0,0005	0,0013	0,0017	0,0020	0,0022
			Total Effect from Activity A_1	**0,0018**	**0,0054**	**0,0074**	**0,0087**	**0,0090**
A_6	H_1	**0,0241**	Direct Effect from Activity A_6	*0,0096*	0,0003	0,0004	0,0005	0,0006
			Indirect Effect from other Activities	*0,0042*	0,0001	0,0002	0,0002	0,0002
			Total Effect from Activity A_6	***0,0138***	**0,0004**	**0,0006**	**0,0007**	**0,0008**
A_6	H_{10}	**0,4183**	Direct Effect from Activity A_6	0,0021	0,0060	0,0081	0,0096	0,0112
			Indirect Effect from other Activities	0,0009	0,0022	0,0030	0,0035	0,0039
			Total Effect from Activity A_6	**0,0030**	**0,0082**	**0,0112**	**0,0131**	**0,0150**

Table 6 (continued) - **Decomposition of m_{ij}: *i* =H_1 and H_{10} and *j* = A_1 and A_6**

Acco-unt j	**Acco-unt i**	**m_{ij}**		**H_6**	**H_7**	**H_8**	**H_9**	**H_{10}**	**Total Househ.**	**% Direct Effect on Account i**
A_1	**H_1**	**0,0154**	Direct Effect from Activity A_1	0,0005	0,0007	0,0005	0,0008	0,0019	0,0115	*38,53%*
			Indirect Effect from other Activities	0,0001	0,0002	0,0002	0,0003	0,0005	0,0039	*13,95%*
			Total Effect from Activity A_1	**0,0006**	**0,0009**	**0,0007**	**0,0011**	**0,0024**	**0,0154**	**52,48%**
A_1	**H_{10}**	**0,3345**	Direct Effect from Activity A_1	0,0087	0,0132	0,0089	0,0157	*0,1880*	0,2590	*56,20%*
			Indirect Effect from other Activities	0,0028	0,0040	0,0034	0,0048	*0,0527*	0,0755	*15,67%*
			Total Effect from Activity A_1	**0,0115**	**0,0172**	**0,0124**	**0,0205**	***0,2407***	**0,3345**	**71,97%**
A_6	**H_1**	**0,0241**	Direct Effect from Activity A_6	0,0007	0,0009	0,0009	0,0011	0,0018	**0,0169**	*39,68%*
			Indirect Effect from other Activities	0,0003	0,0004	0,0003	0,0005	0,0009	**0,0072**	*17,48%*
			Total Effect from Activity A_6	**0,0010**	**0,0013**	**0,0013**	**0,0015**	**0,0027**	**0,0241**	**57,16%**
A_6	**H_{10}**	**0,4183**	Direct Effect from Activity A_6	0,0135	0,0176	0,0177	0,0201	*0,1780*	0,2837	*42,57%*
			Indirect Effect from other Activities	0,0047	0,0069	0,0061	0,0084	*0,0948*	0,1345	*22,66%*
			Total Effect from Activity A_6	**0,0182**	**0,0245**	**0,0238**	**0,0285**	***0,2728***	**0,4183**	**65,22%**

Estimation of Poverty Rates for the Italian Population classified by Household Type and Administrative Region[1]

Claudio Ceccarelli[2], Enrico Fabrizi[3], Maria Rosaria Ferrante[4], Silvia Pacei[5]

Summary

The aim of the work is to provide estimates of some poverty rates for domains defined by cross-classifying the Italian population by household typology and administrative region, on the basis of data collected for Italy by the new "European Union – Statistics on Income and Living Conditions" survey (EU-SILC). This survey is designed to provide reliable estimates for large areas within countries much bigger than the sub-populations of our interest. To solve this problem, we suggest small area estimators derived from multivariate area level models, that improve the reliability of estimates "borrowing strength" over areas and by exploiting the correlation between the considered indicators. The unemployment rate calculated by household typology within administrative regions is used as auxiliary information to improve the precision of model based estimators. As estimation method we use a Hierarchical Bayesian approach implemented by means of MCMC computation methods. Among the different models being compared, the Multivariate Normal-Logistic model is found out to be the best performer.

Keywords: Financial Poverty Measures, European Union - Statistics on Income and Living Conditions, Multivariate Hierarchical Bayes Model

1. Introduction

Fighting poverty and social exclusion has been one of the declared objectives of the European Union since the Lisbon Summit of March 2000 and is increasingly recognized as a major challenge for the international community (European Commission, 2005a). Poverty and social exclusion are distributed unevenly both geographically and across social groups. As a consequence design, implementation and monitoring of effective anti-poverty policies requires data at the level of the relevant or target sub-populations.

Many studies have shown a strong correlation between poverty and some characteristics of the household, namely its composition, with some of the household types markedly more

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exposed to the risk of poverty and social exclusion than others (Christopher *et al.*, 2002; Eurostat, 2005a; 2005b). With reference to Italy the disparities among household types interact with those among the different regions of the country which is characterized by a low degree of regional cohesion (European Commission, 2005a), big differences in regional employement and unemployment rates and high concentration of industrial districts in some geographical areas.

For these reasons, we focus on the estimation of three different poverty rates for domains defined cross-classifying the Italian population by household type and administrative region. Estimates are based on the data collected for Italy by the new "European Community – Statistics on Income and Living Conditions" survey (EU-SILC – 1st wave, year 2004). The three poverty rates are based on different poverty thresholds that are all defined as fractions of the median of the equivalized disposable income, so to distinguish between very poor people, poor people and people who are at risk of becoming poor (Istat, 2007).

The EU-SILC survey is a rotating sample survey on households' income and social conditions, coordinated by Eurostat (Eurostat, 2005c), which allows for the consistent estimation of income distribution parameters and poverty indicators across most of the member states. This survey was designed to provide reliable estimates of main parameters of interest for large areas within countries that, obviously, are much bigger domains (sub-populations) than those we target. The number of units sampled from the domains we consider in this paper is too low, in many cases, to obtain reliable estimates by direct estimators, that is applying standard design-consistent estimators to the domain-specific portions of the sample.

To solve this problem a small area estimation (SAE) strategy is required. In the SAE context linear mixed models are a very common tool (see Rao, 2003, Ch. 5). These models are often based on the assumption that the direct survey estimates and parameters to be estimated are normally distributed. These assumptions may be inappropriate when the support of parameters to be estimated is restricted to the range [0,1] as in the cased of proportions and rates, especially when the true parameter value is close to 0 or 1. As a solution, we suggest the use of two models for small area proportions based on alternative distributional assumptions and compare them with the models based on normality. Moreover, since we are interested in the joint estimation of three different poverty rates we propose the use of multivariate models, that with respect to univariate models have the merit of exploiting the sampling correlation between the direct estimators of the different parameters to improve the precision of small area estimators. Multivariate normal models have been already proposed in the small area literature (Ghosh *et al.*, 1996; Datta *et al.*, 1998). Here we consider multivariate non-normal models. The performances of the SAE estimators proposed in this paper will be compared against those based on the popular Fay-Herriot model (Fay and Herriot, 1979), which may considered to some extent as the "industry standard".

Broadly speaking, a key element to the success of a SAE method is the availability of good auxiliary information. In the case of our application, since Census related or Administrative data are either not available yearly or not fully reliable, we use the estimates of the average annual unemployment rate. Since estimates of the unemployment rate for the domains of interest are not among those routinely published, they have been calculated for the specific purpose of this research by the Italian National Institute of Statistics (ISTAT).

As far as estimation is concerned, we adopt a Hierarchical Bayesian approach implemented by means of MCMC computation methods. It is preferred to the frequentist prediction approach since it allows the handling of complex models such as the multi-level, multivariate non-normal models we consider in a simpler way; in particular we may use posterior variances as natural measures of uncertainty associated to point estimates, while frequentist MSEs will be, for our models, very difficult to obtain. It may also be noted that posterior means and posterior variances enjoy, for careful choices of the prior, good frequentist properties as point estimators and measures of their variability.

The results obtained allow us to compare the incidence of poverty by household type in the different Italian administrative regions. The suggested approach may be extended to the estimation of other indicators and could be used with data collected by the EU-SILC in other countries.

The paper is organized as follows. In Section 2 we briefly review EU-SILC survey. In Section 3 we derive directs estimates and evaluate their reliability. Section 4 introduced the suggested small area models and Section 5 is devoted to the evaluation of the performance of their respective estimators. Conclusions and possible future works are described in Section 6.

2. The “European Community Statistics on Income and Living Conditions” survey

During the period 1994-2001 the European Community Household Panel (ECHP) has been the primary source of data used for the calculation of indicators in the field of Income, Poverty & Social Exclusion for countries of the European Union. Given the need to update the content of the ECHP and in order to satisfy new political demands, to reflect evolving best practice and to improve operational quality, it was decided to replace the ECHP with a new survey, the EU-SILC (European Community Statistics on Income and Living Conditions) (European Parliament and Council, 2003; Eurostat, 2005b). The EU-SILC project was launched in 2004 in Italy.

The aim of the survey is to collect timely and comparable cross-sectional and longitudinal multidimensional microdata on income, poverty, social exclusion and living conditions.

The instrument aims to provide two types of data:

- Cross-sectional data pertaining to a given time or a certain time period with variables on income, poverty, social exclusion and other living conditions;
- Longitudinal data pertaining to individual-level changes over time, observed periodically over, typically, a four years period.

Social exclusion and housing conditions information is collected at the household level while labour, education and health information is obtained for persons aged 16 and over. The core of the instrument, income at very detailed component level, is mainly collected at personal level but a few components are included in the household part of EU-SILC questionnaire.

The number of annual EU-SILC target primary variables is much lower than the number of variables recorded in ECHP (although countries are of course free to include additional variables in their national surveys). The main variables, such as the total household gross and disposable income and the different income components, were redefined to follow as closely as possible the international recommendations of the UN ‘Canberra Manual’ (Eurostat, 2004, 2005b).

A minimum effective sample size is fixed for both longitudinal and cross-sectional components. Sample size for the cross-sectional component is fixed according to a minimum level of reliability for the national estimate of the at risk of poverty rate. The minimum effective sample size assigned to Italy is 7,250 households. Starting from that minimum sample size, the effective sample size is increased taking into account of the chosen sample design (a two-stage design with primary sampling units, that are the Italian municipalities, stratified according to the administrative region and the demographic dimension, and secondary sampling units corresponding to the households) and of the expected response rates (Istat, 2007). Thus, in the first wave (2004) the effective sample size was 32,000 households, 8,000 households for each longitudinal sub-sample.

We know that the sample size allows for the reliable estimation of the average of the net household income at the level of administrative regions (Istat, 2007), but our target domains are more restricted and we wish to estimate a rate whose estimator has a variability is greater than that of the mean. Thus we have, first of all, to face the problem of evaluating the reliability of direct estimates of the poverty rates of interest for our domains.

3. Direct estimates and their reliability

For each domain of interest we want to estimates the following poverty rates:

1. The ‘poverty rate’ (PR) defined the share of persons with an equivalent disposable income below the 60% of median of personal equivalent income (standard poverty threshold).

2. The ‘high poverty rate’ (PR80) defined as the share of persons with an equivalent disposable income below the 80% of the standard poverty threshold.

3. The ‘at risk of poverty rate’ (PR120) defined as the share of persons with an equivalent disposable income below the 120% of the standard poverty threshold.

Personal equivalent disposable income is obtained by dividing total disposable household income by equivalent household size calculated according to the OECD scale commonly used by the Eurostat (it gives a weight of 1.0 to the first adult, of 0.5 to the other persons aged 14 or over in the household and of 0.3 to children under the age of 14). The same equivalent disposable income is assigned to each person in the household.

The domains of interest are 180, obtained cross-classifying the population of the 20 Italian administrative regions by the 9 household typologies considered in the EU-SILC survey; they are defined by simultaneously considering the household size, the presence of children and the age of components. They are defined as follows: 1. One person households; 2. Two adults, no dependent children, both adults under 65 years; 3. Two adults, no dependent children, at least one adult 65 years or more; 4. Other households without dependent children; 5. Single parent household, one or more dependent children; 6. Two adults, one dependent child; 7. Two adults, two dependent children; 8. Two adults, three or more dependent children; 9. Other households with dependent children.

To obtain the direct estimates we use the first EU-SILC wave (2004) whose data are provisional[6].

[6] The reason why the data are still provisional is that, in the case of Italy, in the second wave (2005) important methodological innovations regarding the collection of data and the calculation of aggregates were introduced with respect to the first wave (2004). Hence results from the two waves were not comparable and the first wave of data had to be revised according to the new methodology. First wave definitive data were not available when this work started. Nevertheless, the provisionalness of data do not influence the validity of the results presented in this work.

Since the domain we consider are not planned, we modified the official final weights published in the EU-SILC data set in order to have weights calibrated on the distribution of the Italian population by administrative region and household typology. Final published weights are obtained by a double calibration correction. The first step adjusts basic weights (i.e. those obtained as inverse of the inclusion probabilities) for non-response, while the second step modifies these intermediate weights to calibrate them to known totals as suggested in the Eurostat guidelines for the EU-SILC survey (Istat, 2007). In particular the distribution of population by gender, age class and geographical region is considered.

To obtain weights calibrated on the distribution of the population in the domain of interest (i.e. administrative region by household type) we start from the survey intermediate weights are re-make the second step, considering the following calibration variables: administrative region of residence; household type; gender; age (5 classes). Totals are obtained from the same data sources used in the derivation of final official weights for all variables except the distribution of the population by household type within administrative regions which has been obtained as average of the quarterly Labor Force Survey results obtained in 2004. In the calculation of the calibration weights, the log distance, leading to raking-ratio weights is used: it has the advantage of producing always positive weights.

To evaluate the reliability of those estimates we then need to estimate their variances and, to apply the small area multivariate models presented in the next section, we need also to estimate the covariances between estimates of different rates obtained for the same domain. Evaluating the variance and covariance of the direct estimators is in this case a complicated task, as i) the considered poverty indicators are complicated functions of data; ii) the underlying design is complex; iii) the weights used in their computation incorporate, as it has been previously described, two stages of calibration corrections.

In keeping with other work in this field (Verma and Betti, 2005), we opt for a solution based on re-sampling algorithms and in particular we propose a bootstrap estimation strategy. Bootstrap variance estimators have been proposed and analyzed for sampling designs as general as the multi-stage sampling design with stratification of primary units. See Rao (1999) for more details. These estimators rely on the assumptions that the number of strata is large and that few primary units (but at least two) are sampled from each stratum, so that the sampling fraction at the first stage is negligible. This latter assumption is not met in our case as there exists a stratum of self-representative municipalities (primary units) that are always included into the sample. For this reason, we propose a bootstrap algorithm in which any bootstrap sample is the union of two sub-samples, one taken resampling the population in the non self-representative strata and the other drawn from the self-representative stratum, where the sampling design is actually single stage. After it is drawn from the population each bootstrap sample undergoes the same calibration adjustment of weights to known totals applied to the original sample. The algorithm has been tested by means of simulation exercises and, found to provide estimates close to those obtained using the linearization method for parameters for which the latter method can be applied.

Another problem is due to the absence of poor persons in the sample of some domains. This would lead to an estimate of the correspondent poverty rate equal to zero in the domain, so and to zero estimates of variance and covariance. For the moment we avoid that problem by discarding those domains (15), so to postpone its solution to future work.

As the number of domains is too high to present results obtained for each of them, we present, as usual, summary measures, that are indicators allowing us to evaluate i) the variability of estimates between the domains; ii) their reliability, through synthesis made with respect to the set of the domains.

We denote with $\hat{\theta}_{ij}$ the direct estimate of θ_{ij}, the poverty rates $(PR80, PR, PR120)$ in the *i,j*-th domain, where *i* denotes the region $(i = 1,...,20)$ and *j* the household type $(j = 1,...,9)$. In table 1 value for minimum, maximum, average, median and skewness for $\hat{\theta}_{ij}$ and also minimum, maximum and average of their coefficient of variation for each poverty rate considered are reported.

Table 1 - Summary of results obtained for the direct estimates

	Parameter (in %)		
	PR80	PR	PR120
$\min\left(\hat{\theta}_{ij}\right)$	0.003	0.021	0.057
$\max\left(\hat{\theta}_{ij}\right)$	0.592	0.683	0.733
$\text{avg}\left(\hat{\theta}_{ij}\right)$	0.117	0.207	0.310
$\text{median}\left(\hat{\theta}_{ij}\right)$	0.074	0.172	0.292
$\min\left[CV\left(\hat{\theta}_{ij}\right)\right]$	0.111	0.079	0.061
$\max\left[CV\left(\hat{\theta}_{ij}\right)\right]$	1.398	0.837	0.581
$\text{avg}\left[CV\left(\hat{\theta}_{ij}\right)\right]$	0.422	0.280	0.194

From Table 1 we may note that there are big differences among the domains in terms of poverty rates, as well as in terms or reliability of estimators.

As regards the reliability of direct estimators, the coefficients of variations are, on average, too high to consider the direct estimates sufficiently reliable, even for the case of PR120 for which the average is about 20%. Therefore a small area estimation strategy is needed.

In Table 2 the average correlations between the three set of rates are displayed: as expected they are positive and far from 0: this justifies the recourse to multivariate models.

Table. 2 - Estimated correlation matrix (data average over the domains)

	PR80	PR	PR120
PR80	1	0.7	0.51
PR		1	0.73
PR120			1

4. The considered small area models

A general SAE area level model consists of two parts, a "sampling model" for the sampling errors of the direct survey estimators and a "linking model" that relates parameters to area specific auxiliary information. Here we propose to use estimators based on multivariate area level models that borrow strength not only from other areas but also from the sampling correlation between survey estimates of different parameters.

Let $\boldsymbol{\theta}_{ij} = \left(\theta_{ij1},...,\theta_{ijk},...,\theta_{ijK}\right)^T$ be the vector of $K = 3$ parameter of interest for the *ij*-th domain ($i = 1,...,m = 20$ and $j = 1,...,J = 9$) and $\hat{\boldsymbol{\theta}}_{ij}$ the corresponding vector of survey estimates. $\boldsymbol{\theta}_{ij}$ and $\hat{\boldsymbol{\theta}}_{ij}$ are linked by the following sampling model:

$$\hat{\boldsymbol{\theta}}_{ij} \,\Big|\, \boldsymbol{\theta}_{ij} \sim N_K\left(\boldsymbol{\theta}_{ij}, \boldsymbol{\Psi}_{ij}\right) \tag{4.1}$$

where the $K \times K$ positive definite $\boldsymbol{\Psi}_{ij} = V\left(\hat{\boldsymbol{\theta}}_{ij} \mid \boldsymbol{\theta}_{ij}\right)$ is assumed to be known and equal to the estimate obtained according to the bootstrap method illustrated in previous section (see Rao 2003, p. 76).

The first linking model we consider is based on the usual assumption of normality:

$$\boldsymbol{\theta}_{ij} \,\Big|\, \boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}_v \sim N_K\left(\boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}_v\right) \tag{4.2}$$

where $\boldsymbol{\Sigma}_v$ is an assumed positive definite $K \times K$ prior variance matrix, $\boldsymbol{\mu}_{ij} = \left(\mu_{ijk}\right)_{k=1,...,K}$ is a *K*-dimensional vector with the elements defined as:

$$\mu_{ijk} = \alpha_k + \tilde{\mathbf{x}}_{ijk}^T \boldsymbol{\beta}_{jk} \tag{4.3}$$

$\boldsymbol{\beta}_{jk}$ being a $p \times 1$ vector of regression coefficient and $\tilde{\mathbf{x}}_{ijk}$, the $p \times 1$ vector of auxiliary data. Let α_k be a rate specific intercept.

The matching of [4.1] and [4.2] produces a linear mixed model. We refer to this as to a Multivariate Normal-Normal model (M-NN). Similar models are considered in Datta *et al.* (1999), in Ghosh *et al.* (1996) and in Fabrizi *et al.* (2008).

Unfortunately the normality assumption does not guarantee that the estimates of rates fall into the [0,1] interval. In the following we consider two alternative linking models, alternative to the M_NN one, suitable for rates.

At first we choose a $\text{logit}(\boldsymbol{\theta}_{ij})$ linking model:

$$\text{logit}(\boldsymbol{\theta}_{ij}) \,\Big|\, \boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}_v \sim N_K\left(\boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}_v\right) \tag{4.4}$$

where:

$\text{logit}\left(\boldsymbol{\theta}_{ij}\right) = \left\{\text{logit}\left(\theta_{ijk}\right)\right\}_{k=1,...,K}$, and μ_{ijk} defined as in [4.3].

As the sampling and the linking model cannot be combined into a single expression, we say that this model is unmatched in the sense of You and Rao (2002). The logit linking model [4.4] has already been considered in the SAE context (Farrel *et al.,* 1997; Malec *et al.*, 1999; Liu *et al.*, 2007) but in the univariate form.

We refer to the SAE model based on [4.1] and [4.4] as to the Multivariate Normal-Logistic model (M_NL).

We propose a second linking model based on the Beta distribution assumption and on the logit link too, having the property of producing estimates in the [0,1] range:

$$\begin{aligned} &\theta_{ijk}\big|\mu_{ijk},\phi_k \sim Beta\big(\mu_{ijk},(1-\mu_{ijk})\phi_k\big), \quad \text{with } E(\theta_{ijk})=\mu_{ijk} \\ &\text{logit}(\boldsymbol{\mu}_{ij})\big|\boldsymbol{\mu}_{ij},\Sigma_v \sim N_k\big(\boldsymbol{\mu}_{ij},\Sigma_v\big) \end{aligned} \tag{4.5}$$

We refer to this model as Multivariate Normal-BetaLogistic (M_NBL).

Finally, as a benchmark, we consider also a standard Univariate Normal-Normal model (U_NN) defined for each rate:

$$\begin{aligned} &\hat{\theta}_{ijk}\,\big|\,\theta_{ijk},\psi_{ijk} \sim N\big(\theta_{ijk},\psi_{ijk}\big) \\ &\theta_{ijk} \sim N\big(\mu_{ijk},\sigma_k^2\big) \\ &\mu_{ijk} = \alpha_k + \tilde{\mathbf{x}}_{ijk}^T\boldsymbol{\beta}_{jk} \end{aligned} \tag{4.6}$$

In the following the prior specification for the described models is reported:

$$\begin{aligned} &\alpha_k \sim N(0,a_1) \\ &\boldsymbol{\beta}_{jk} \sim N(0,a_2\mathbf{I}_p) \\ &\boldsymbol{\Sigma}_v^{-1} \sim Wishart(\mathbf{I}_k,K) \\ &\phi_k \sim N(0,a_3)I(\phi_k \geq 0) \end{aligned}$$

where a_1, a_2 and a_3 are large with respect to the scale of data, thus defining “diffuse proper priors”. For the univariate normal model [4.6] we consider the following prior for the variance component: $\sigma_k \overset{ind}{\sim} Unif(0,a_4)$ following the suggestion of Gelman (2006). Again a_4 is set in order to minimize the impact of prior specification on posterior summaries.

As anticipated in the introduction we consider a single covariate $(p=1)$: the *average annual unemployment rate*, defined for each domain and obtained from the Italian Labor Force Survey. The covariate is the same for each of the parameters of interest because of its good or acceptable predictive power in each case (R^2 ranging from 0.54 to 0.58) and selected as the best among different labor force rates.

The approximations of the posterior distributions are computed using the WinBugs software (Spiegelhalter *et al.*, 2003) which is very widely used in applied hierarchical modeling.

5. The small area estimators performance

In this section we evaluate the adequacy of the proposed models and the gains in efficiency obtained with predictors associated to the discussed models with respect to the direct estimators.

In the posterior predictive assessment approach, to check how well a given model fits the data, new observations are generated according to the posterior distribution of the given model. If the fit is adequate, then the generated observations should be similar to the observed data. To quantify the discrepancy between newly generated and observed data many possible discrepancy measures may be used: we consider the following one proposed in Datta *et al.* (1999):

$$d\left(\hat{\theta}_k, \theta\right) = \sum_{i,j} \psi_{ij(kk)}^{-1} \left(\hat{\theta}_{ijk} - \theta_{ijk}\right)^2$$

where $\psi_{ij(kk)}$ is the *k*-th diagonal element of $\mathbf{\Psi}_{ij}$ matrix. On the basis of this discrepancy measure we can calculate the posterior predictive *p-values* as the probability that the discrepancy measure calculated for the generated new data is larger than that obtained for the observed data, given the observed data. The posterior predictive *p-value*, is expected to be near 0.5 if the model adequately fits the data.

Moreover, the suggested models are compared on the basis of the deviance information criterion (DIC), a hierarchical modelling generalization of the AIC (Akaike information criterion) and BIC (Bayesian information criterion). It is particularly useful in Bayesian model selection problems where the posterior distributions of the models have been obtained by Markov chain Monte Carlo (MCMC) simulation (Spiegelhalter *et al.*, 2002). The model with the smallest DIC is assumed to be the model that would best predict a replicate dataset which has the same structure as that currently observed. In Table 3 values obtained for such measures for the considered models are reported.

Table 3 - Bayesian measures of model fit

	U_NN	M_NN	M_NL	M_NBL
	p-value			
PR80	0.02	0.54	0.72	0.75
PR	0.19	0.53	0.53	0.51
PR120	0.32	0.54	0.35	0.41
DIC	-1459	-1717	-1902	-1845

Based on the *p-values* the fit of the model results to be adequate on average for all three multivariate models, whereas U_NN clearly shows a lack of fit. Looking at DIC values, the best model seems to be the M_NL one, followed by M_NBL.

As regards the gain in efficiency, we consider: *i*) the average percentage reduction of the Coefficients of Variation of the small area estimators versus the direct one; *ii*) a measure of coverage that is the width of the 95% credible interval for the mean.

The reduction of the Coefficient of Variation of a small area estimator versus the direct ones is measured as follows. Let $\hat{\theta}_{ijk}^{h}$ and $\hat{\theta}_{ijk}^{D}$ be respectively the small area estimate obtained from model h $\left(h = U\_NN, M\_NN, M\_NL, M\_NBL\right)$ for parameter k $\left(k = PR80, PR, PR120\right)$ and the direct estimate for parameter k. As measure of the improvement precision we use the percent reduction of the Coefficient of Variation realized by the $\hat{\theta}_{ijk}^{h}$ versus the $\hat{\theta}_{ijk}^{D}$, evaluated on average on areas ($ACVR_{kh}$):

$$ACVR_{kh} = 100 - \frac{ACV_{kh}}{ACV_{kD}} 100$$

where $ACV_{kh} = \frac{1}{mJ} \sum_{ij} \frac{\sqrt{MSE\left(\hat{\theta}_{ijk}^{h}\right)}}{\hat{\theta}_{ijk}^{h}}$ (same for ACV_{kD} based on $\hat{\theta}_{ijk}^{D}$).

Moreover, in order to evaluate the shrinkage effect, typically connected to small area estimators (see Rao, 2003 p. 211); a measure of the reduction of the variability of estimates between domains is provided. This indicator is based on the ratio of the variance between areas of $\hat{\theta}_{ijk}^{h}$ versus the variance of the direct estimator $\hat{\theta}_{ijk}^{D}$. The Variance Reduction based on the Ratio of the variance between areas of θ_{ijk}^{h} versus the variance of the direct estimator $\hat{\theta}_{ijk}^{D}$:

$$VARR_{kh} = 100 - 100 \frac{Var\left(\hat{\theta}_{ijk}^{h}\right)}{Var\left(\hat{\theta}_{ijk}^{D}\right)}$$

Table 4 contains the results obtained for these three measures for the parameters being estimated.

Table 4 - Estimators performance in terms of gain in efficiency, shrinkage, coverage (average measures)

	PR80	PR	PR120
	Average Coefficient of Variation reduction (%)		
U_NN/D	30,47	24,77	21,47
M_NN/D	10,49	12,43	11,51
M_NL/D	27,96	17,39	12,53
M_NBL/D	22,29	14,41	11,37
	Average Variance Reduction (%)		
U_NN/D	41,77	20,82	12,27
M_NN/D	23,64	18,43	14,38
M_NL/D	19,60	19,69	17,59
M_NBL/D	19,31	19,08	16,97
	Mean credible interval width		
U_NN	0,081	0,131	0,157
M_NN	0,120	0,157	0,178
M_NL	0,109	0,150	0,171
M_NBL	0,113	0,154	0,174

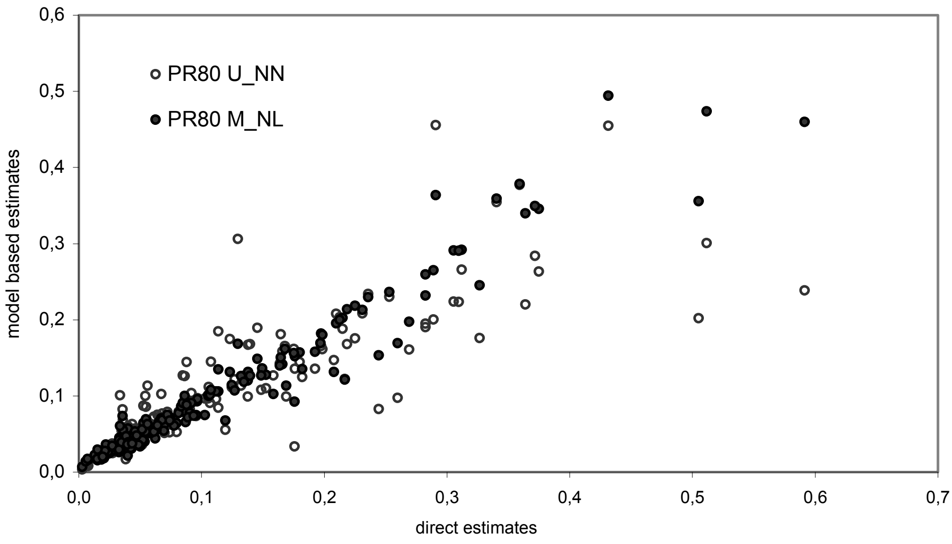
We may observe that estimators derived from U_NN model perform better than the estimators derived from the multivariate models in terms of gain of efficiency: by the way predictors based on this model over-shrink a lot, reaching a unacceptable level of average variance reduction with respect to the direct estimator especially for PR80 (about 42%).

Among the multivariate models, the most significant gain in efficiency is associated to the M_NL model, even though M_NBL leads to similar results, especially for PR120. We may note that the reduction of CV driven by the M_NL on PR80, for which the direct estimates showed the highest CVs (Table 1), is noticeable (about 28% vs. the 10% of the M_NN) and that it leads to a shrinkage that is comparable to that induced by other models, and consistent with results obtained in another study based on simulations and carried out on ECHP data (Fabrizi *et al.*, 2007). In fact in that work it found that the direct estimators tend to be over-dispersed with respect to the true domain parameters of about 10% and the considered small area estimators (EBLUP derived from Liner Mixed Models) tend to reduce the variance between domains of the true parameters of 10%.

Moreover, the M_NL model produces also the smallest average interval width, while for M_NN, the 20% of the credible intervals has negative lower bound on PR80 and in some cases also in PR.

We evaluate also the performance of the suggested models by considering the estimates obtained for each domain. In the following, for sake of brevity, we only illustrate the results that we think as the most interesting among those obtained.

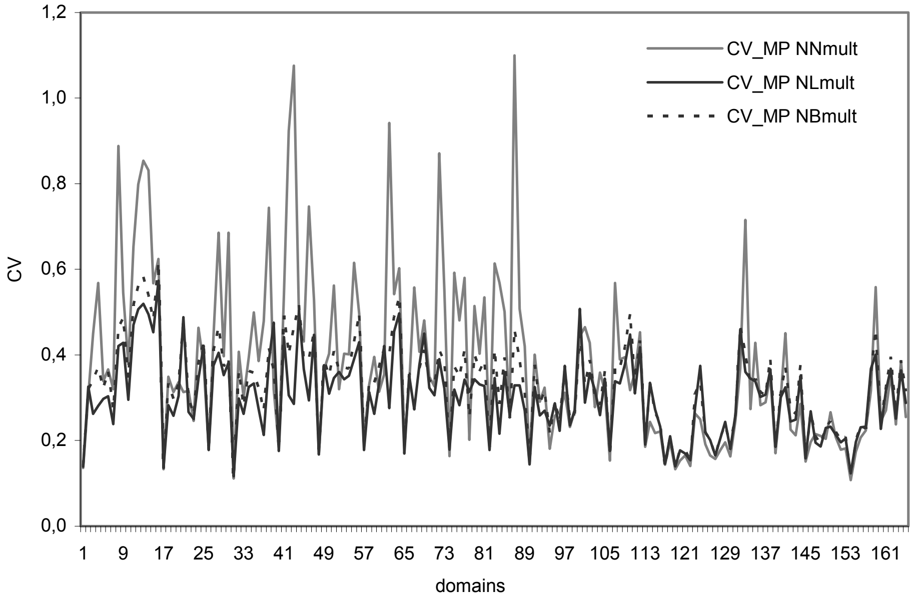
Fig. 1 - Direct estimates vs. U_NN and M_NL estimates (PR80)



From fig. 1, referred to PR80, it turns out that model based estimates obtained by using the model previously singled out as the most performing, M_NL, are closer to the direct estimates that those derived by the U_NN model. In fact, for a noticeable number of domains this last produces estimates that are the much more high or much more low than the corresponding direct estimates.

In fig. 2, where we compare the CVs of the estimates produced by M_NB, M_NL and M_NN, it can be seen that the CV associated to this latter model assume in a large number of domains very high values (in the 22% of domains, CV is higher that 50%). On the contrary, the most part of CV produced by M_NB and, mostly, by M_NL, are acceptable: for the 25% of domains CVs are higher than 30%.

Fig. 2 - Coefficient of variation, M_NN, M_NL and M_NB estimates (PR80)



6. Concluding remarks and future work

Although they are very simple and give only a partial picture of poverty, poverty rates are essential descriptive quantities in the study of social exclusion. In this paper we devise a methodology in which the use of resampling algorithms, auxiliary information and model based estimators may be integrated to produce reliable estimates for small sub-population that may be of key interest to social scientists and policy makers. By the way, some problems are left open.

In fact, in this research we considered alternatives to the normality assumption in the linking part of the model; but it may noted that also the normality of the direct estimators, that may be justified on the basis of a central limit argument is questionable. One possible direction for further research is the implementation of a Beta sampling model similar to that considered by Jiang and Lahiri (2006) in univariate form, in connection with the linking models here used.

In some domains direct estimates are equal to zero, this giving sampling variances equal to zero and this does not necessarily imply a high degree of accuracy of the estimates (Elazar, 2004; Ghosh and Maiti, 2004). To deal with the zero variances problem, the solution adopted in literature is to discard the estimates equals to zero: the aim is to propose a smoothing covariance matrix solution, in a Generalized Covariance Function approach.

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